

Each question is 20 points. You are supposed to work without the help of other students! Please justify all answers.

1. Consider the vector space V of all functions $\mathbb{R} \rightarrow \mathbb{R}$, and its three ‘vectors:’ $f(x) = \sin^2(x)$, $g(x) \equiv 5$, $h(x) = \cos(2x)$. Are these three vectors linearly independent? Why or why not?
2. Let

$$A = \begin{bmatrix} 5 & -4 & 4 \\ 2 & -1 & 2 \\ -4 & 4 & -3 \end{bmatrix}.$$

Compute the determinant, the eigenvalues, and the characteristic polynomial for A . What is the connection between the last two items?

3. Find a basis for the set of vectors in \mathbb{R}^3 in the plane S , where S is given by $3x + 5y - 7z = 0$.
4. Let A be an $m \times n$ matrix. Prove or disprove the following statement: The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b} \in \mathbb{R}^m$ if and only if the equation $A^T\mathbf{x} = \mathbf{0}$ has only the trivial (i.e. zero) solution. (Here A^T denotes the transposed of A .)

Good luck!