Introduction to Probability and Statistics (3510)
Department of Mathematics
University of Colorado, Boulder

Final Exam (100 points)

Date: May 3, 2010
150 minutes

Rules:

1. The final exam is closed book/notes.
2. You should write **all your work** on the exam, there is sufficient space provided below each question.
3. Using a calculator is OK, but using a cell phone, a laptop, or an iPad is not. In general, you **cannot use any communication device** for calculations.
4. Write your student ID below if available, and of course, your name.
5. Try to write in a **legible** way – it lowers the chance that your correct work is misinterpreted.
6. At the end of the exam you will find the necessary formulas (and perhaps some extra formulas too), and the tables.
7. Justify your answers. A correct numerical answer without justification worth less than a correct argument with a missing or erroneous numerical answer.

- Your name: _____
- Your ID: _____

Questions start on next page. **GOOD LUCK!**

1. [15p] We took a sample of 6 from a normal distribution with unknown mean and standard deviation. We got the values 7.9, 8.1, 8.2, 7.8, 7.9, 8.2. Can we be 90% certain that the mean is within distance 0.1 from our point estimate? Why?

2. [15p] In seeking to determine the proportion of snowy days in Boulder, we check 50 days. Assuming that there is no connection between weather on different days, at what confidence level can we assure that the error in the reported proportion of snowy days is less than 0.1 ?

3. [20p] Suppose that x is a value of a random variable having uniform distribution on the interval $[0, \lambda]$. We do not know the value of λ but we know that it is less than 5. Find an $r > 1$ so that $[0, rx]$ is a $(1 - \alpha)100$ percent confidence interval for the mean. Your answer should not include λ but it may include α.

4. [15p] Your friend says that after tossing a fair coin 1000 times, she got less than 450 heads. Can we believe this? Why?

5. [10p] The number of large forest fires in Colorado is different in every year but we know that its variance is 4.5. What is the probability that next year there will be no large forest fire at all in CO?

6. [25p] We have a box, in it there are three white balls and two blacks. We toss a die which is biased: the probability of heads is 2/3. If we get heads, then we draw 3 balls *with* replacement, and if it is tails, then 3 balls *without* replacement. What is the probability that we get *exactly* one black ball?

Useful formulae

- A useful inequality: $x(1-x) \leq \frac{1}{4}$ for $0 \leq x \leq 1$.
- Estimate for variance:

$$\overline{S}_n^2 = \frac{1}{n-1} \sum_1^n \left(X_i - \overline{X}_i\right)^2.$$

  If n is large, then $n-1$ may be replaced by n, but for small n it cannot.

- Standard deviation for proportions: $\sqrt{p(1-p)}$.
- A version of CLT: .

$$\frac{\sum_1^n X_i - n\mu}{\sqrt{n}\sigma}$$

  is approximately standard normal if n is large (use the cutoff $n = 30$.)

- $P(A \mid B) = \frac{P(AB)}{P(B)}$.