Introduction to Probability and Statistics (3510)
Department of Mathematics
University of Colorado, Boulder

Final Exam (100 points)

Date: Dec 16, 2010
150 minutes

Rules:

1. The final exam is closed book/notes.
2. You should write **all your work** on the exam, there is sufficient space provided below each question.
3. Using a calculator is OK, but using a cell phone, a laptop, or an iPad is not. In general, you **cannot use any communication device** for calculations.
4. Write your student ID below if available, and of course, your name.
5. Try to write in a **legible** way – it lowers the chance that your correct work is misinterpreted.
6. At the end of the exam you will find the necessary formulas and the tables.
7. Justify your answers. A correct numerical answer without justification worth less than a correct argument with a missing or erroneous numerical answer.

- Your name: _____
- Your ID: _____

Questions start on next page. **GOOD LUCK!**

1. [25p] We throw a die. If we get 1 or 2, then we toss three fair coins; otherwise, we toss four fair coins. What is the probability that the number of heads is precisely two?

2. [10p] The number of earthquakes in Los Angeles is different in every year but our record shows that its average is 3.4. What is the probability that next year there will precisely one earthquake in Los Angeles?

3. [15p] Your friend says that after tossing a hundred fair dice, the sum of the dice was not more than 300. Do you believe this? Why?

4. [15p] The location of a particle (relative to a reference point) in an experiment is a random variable and we know that its distribution is normal. Unfortunately, we do not know the mean and the standard deviation. Running the same experiment five times independently, we made 5 measurements and got the values -3.3, -3.1, 0, -2.8, -3.0 (in nanometers). Can we be 95% certain that the mean is within 0.5 nanometer from our point estimate? Why?

5. [15p] A medical survey aims to determine the proportion of HIV positive individuals in an African city. A hundred randomly selected individuals, living in distant areas in the city, are checked. At what confidence level can we assure that the error in the reported proportion of HIV positive individuals is less than 0.1 ?

6. [20p] Suppose that x is a value of a random variable having uniform distribution on the interval $[0, \lambda]$. We do not know the value of λ. Your friend thinks that the interval $[0, 2x]$ is a 90% confidence interval for the mean of the random variable. Is your friend right? Why?

Useful formulae

- A useful inequality: $x(1-x) \leq \frac{1}{4}$ for $0 \leq x \leq 1$.
- Estimate for variance:

$$\overline{S}_n^2 = \frac{1}{n-1} \sum_1^n \left(X_i - \overline{X}\right)^2.$$

  If n is large, then $n-1$ may be replaced by n, but for small n it cannot.

- Standard deviation for proportions: $\sqrt{p(1-p)}$.
- A version of CLT: .

$$\frac{\sum_1^n X_i - n\mu}{\sqrt{n}\sigma}$$

  is approximately standard normal if n is large (use the cutoff $n = 30$.)

- $P(A \mid B) = \frac{P(AB)}{P(B)}$.