Introduction to Probability and Statistics (3510)
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Solutions to HW1

2.1 Yes. To each face, consider its "pair" (even though now it is not on the opposite side): $1-6, 2-5, 3-4$. Pairs add up to 7. Thus, any particular outcome is as likely as the corresponding other one. So, the probability that the sum is a is the same as the probability that it is $21-a$. Now take $a = 9$.

2.2 Every contestant being equally likely to make the top four: P(Sheila makes top four)= $\frac{\text{nr of good groups}}{\text{nr of all groups}} = \frac{\binom{11}{3}}{\binom{12}{4}} = 1/3$.

Alternatively, $P(\text{Sheila picked first}) = 1/12$, and $P(\text{Sheila picked second}) = (11 \cdot 1)/(12 \cdot 11) = 1/12$, with similar reasoning showing that the probability she is picked third or fourth are also both $1/12$.

2.3 The sample space here is four-letter long strings, each letter being an "M" or "F", for a total of $2^4 = 16$ elements. (Puppies are distinct things, so we take order to matter.)

So $P(2 \text{ "M"´s and } 2\text{"F"´s}) = \frac{\binom{4}{2}}{2^4} = 6/16 = 3/8$, while $P(3 \text{ "M"´s and } 1\text{"F"}) = \frac{\binom{4}{1}}{2^4} = 4/16 = 1/4$, with the probability of 3 "F"'s and 1 "M" the same. Thus, the probability of seeing 3 of one gender and 1 of another is $1/2$ which is bigger than the $3/8$ probability of seeing two of each.

2.4 (a) Each of three people gets two choices (Plumber A or Plumber B) for a total of $23 = 8$ elements in the sample space. There are exactly two ways they can all make the same choice: each picks A or each picks B. So, the probability they all pick the same plumber is $2/8 = 1/4$.

(b) We call R_1, R_2, R_3 the values of the first, second, and third rolls respectively. We wish to compute $P(R_1 < R_2 < R_3)$. We index the possible ways the event of interest can happen via R_2 : If $R_2 = 2$, R_1must be 1, and R_3 could be $3, 4, 5, 6$. If $R_2 = 3, R_1$ could be 1, 2 and R_3 could be 4, 5, 6. If $R_2 = 4$, R_1 could be 1, 2, 3 and R_3 could be 5, 6. If $R_2 = 5$, R_1 could be 1, 2, 3, 4 and R_3 must be 6. We have enumerated 20 elements in the event. Clearly the sample space has $63 = 216$ elements, and the probability is $20/216$.

(c) We need to compute the probability that A wins (and see whether this is 1/2). Now $P(A \text{ wins}) = P(dif = 0) + P(dif = 1) + P(dif = 2)$ where "dif" is the numerical difference between the two rolls. There are 6 ways for dif$= 0$, 10 ways for dif$= 1$ and 8 ways for dif$= 2$. So $P(A \text{ wins}) = 6 + 10 + 836 = 24/36$.

2.5 It makes no difference whether you pick first or last. If you pick first, you may consider your sample space consisting of just 10 options, with just one of them being the winner so P (win, pick first) $= 1/10$. On the other hand if you go last, the sample space now consists of the 10! possible orderings for everyone, including you, to pick. And now, the number of ways that people can pick so that you win is 9! as the 9 people that go before you must choose from the 9 non-winning tickets in any order. Hence $P(\text{win, pick last}) = 9!/10! = 1/10$.

2.13 No, it is not fair. If E denotes "even" and O denotes "odd" then there are four, equally likely possibilities to come out: EO, OE, EE and OO. Only in the first three it is true that at least one of them is odd. But in this case only every third time it will be true that the other one is even as well. So if you bet with odds $1:1$ in these cases, you will lose money.