Introduction to Probability and Statistics (3510)

Solutions to First Midterm (2/20/2012)

Q1 [30 points] A family of four always has dinner at the same dining table which has four chairs. Everyone has his/her own chair but tonight they will sit down completely randomly. What is the probability that there is exactly one family member sitting on his/her own chair?

SOL: Let us fix who the lucky one is. The probability for having, say mom, at the right place and all the others at wrong places is $(1/4) \cdot (2/3!) = 1/12$. This must be multiplied by four, because we do not know who the lucky one is, and we are talking about four disjoint events with the same probabilities. So the final answer is 1/3.

Q2 [20 points] We toss three dice. What is the probability that the sum is a multiple of three?

SOL: Whatever the first two are, there is always 1/3 chance that the third one will give the right remainder with 3. That is, if the sum has remainder 0/1/2, then the third one has to give 0/2/1 as a remainder. So, the final answer is 1/3.

Q3 [30 points] We toss two dice. What is the expected value of the difference between the two numbers? (The difference between a and b is |a - b|.)

SOL: If you draw a 6×6 table for the outcomes, you can see that the pmf for the difference is: 0 has chance 6/36; 1 has chance 10/36; 2 has chance 8/36; 3 has chance 6/36; 4 has chance 4/36 and 5 has chance 2/36. Then use the formula $\sum_{i=0}^{5} i \cdot p_i$.

Q4 [20 points] Let A, B and C be three events. Let D be the event that at least one of them occurs. Express the probability of D, using the probabilities of A, B, C, AB, AC, BC, and ABC. (We use AB as a shorthand for $A \cap B$.)

SOL:

$$P(D) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC).$$