SOLUTIONS TO THE FOURTH HW SET

February 27, 2012

3.9

This is a poorly worded problem: you want to see two cars (car 3 and car 12 say) which both have, for example, first digit 2 and last digit 7. (That is, you are not looking for the chance that one car has both the first and last digit equal to 2 and some other car has first and last digit equal to 7.) That said, this is a birthday problem with 15 "people" and $10 \cdot 10 = 100$ "days". So

$$P(\textit{at least two matches}) = 1 - P(\textit{no matches}) = 1 - \frac{100 \cdot 99 \cdot \ldots \cdot 86}{100^{15}}.$$

■

3.10

There are 37 numbers on the roulette wheel. To ask for some number to come up at least twice in 10 spins is asking the birthday problem with 10 "people" and 37 "days". Thus, the probability asked for is

$$1 - \frac{37 \cdot \ldots \cdot 28}{37^{10}}.$$

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3.11

(a)

Yet another birthday problem. There are seven actual people and now they are labeled by one of 25 possible floors. So

$$P(\textit{at least } 2 \textit{ on the same floor}) = 1 - \frac{25 \cdot 24 \cdot \ldots \cdot 19}{25^7}. \qquad \blacksquare$$

(b)

We still compute through the opposite:
P(someone has the same floor as you) =
1−P(no one has the same floor as you) =$1-(24/25)^6$. ■

3.14

(a)

So there are 10^4 possible winning numbers and 10 drawings. Another birthday problem with 10 "people" and 10,000 "days." So, P(see same winners somewhere) $= 1 - \frac{10000 \cdot \ldots \cdot 9991}{10000^{10}}$. ■

(b)

Let's call the probability computed in the first part p. (We don't care what it is). We repeat the experiment of the first part 300 times (independently) and ask for the probability that the result (seeing the same number in two different states) occurs at least twice. Again, it is easier to compute the probability that it never happens, and subtract this from 1. That is, the desired probability is

$$1 - (1 - p)^{300}.$$

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