Math 2400, Midterm 3

April 15, 2019

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50 AM
☐	Section 002	Harrison Stalvey	8:00–8:50 AM
☐	Section 003	Daniel Martin	9:00–9:50 AM
☐	Section 004	Albert Bronstein	9:00–9:50 AM
☐	Section 005	Xingzhou Yang	10:00–10:50 AM
☐	Section 006	Mark Pullins	10:00–10:50 AM
☐	Section 007	János Englander	10:00–10:50 AM
☐	Section 008	John Willis	12:00–12:50 PM
☐	Section 009	Taylor Klotz	1:00–1:50 PM
☐	Section 010	János Englander	2:00–2:50 PM
☐	Section 011	Harrison Stalvey	2:00–2:50 PM
☐	Section 012	Xingzhou Yang	3:00–3:50 PM
☐	Section 013	Trevor Jack	4:00–4:50 PM

Question	Points	Score
1	12	
2	4	
3	15	
4	15	
5	14	
6	12	
7	15	
8	5	
9	4	
10	4	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 95 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (12 points) Match the vector fields $\overrightarrow{\boldsymbol{F}}$ with the plots below.

(1) $\overrightarrow{\boldsymbol{F}} = \langle y, x \rangle$ ____

(2) $\overrightarrow{\boldsymbol{F}} = \langle x, -y \rangle$ ____

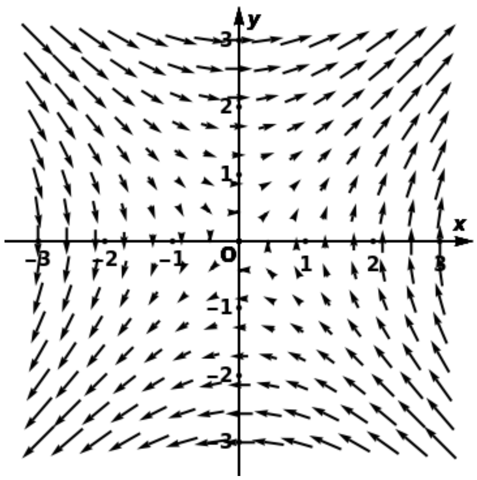
(3) $\overrightarrow{\boldsymbol{F}} = \langle y, -x \rangle$ ____

(4) $\overrightarrow{\boldsymbol{F}} = \left\langle \dfrac{x}{\sqrt{x^2+y^2}}, -\dfrac{y}{\sqrt{x^2+y^2}} \right\rangle$ ____

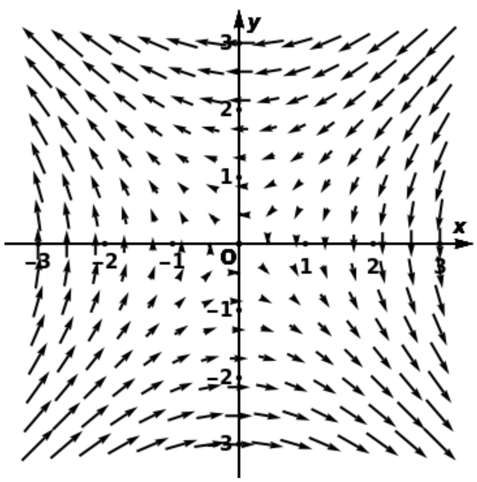
(5) $\overrightarrow{\boldsymbol{F}} = \langle 2, x-y \rangle$ ____

(6) $\overrightarrow{\boldsymbol{F}} = \langle -y, -x \rangle$ ____

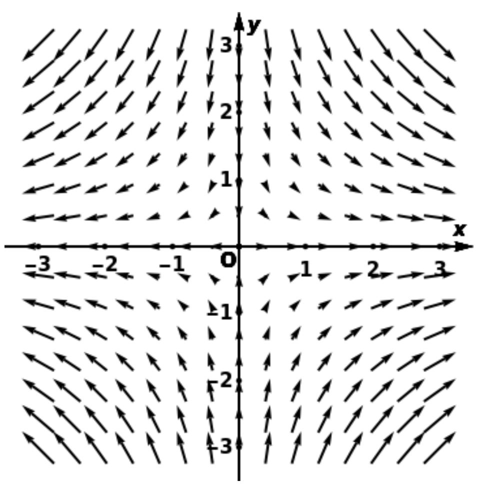
(A)



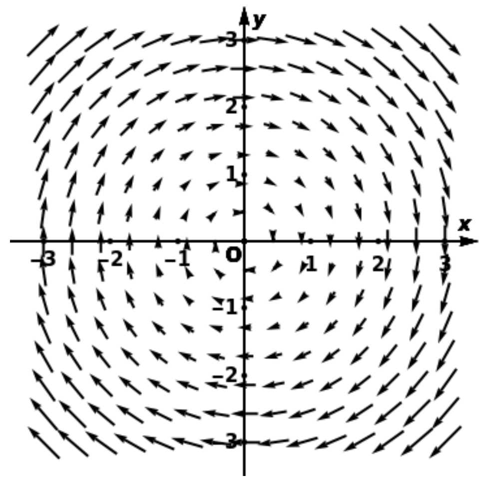
(B)



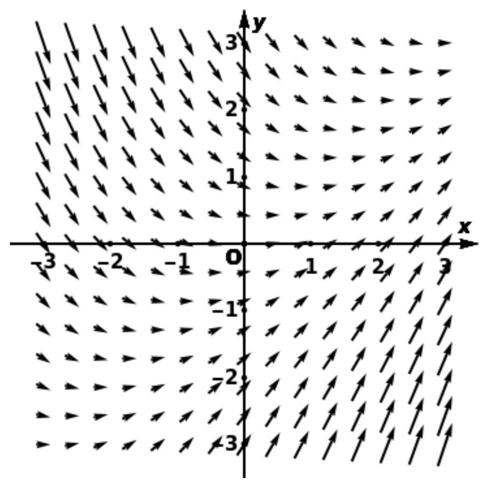
(C)



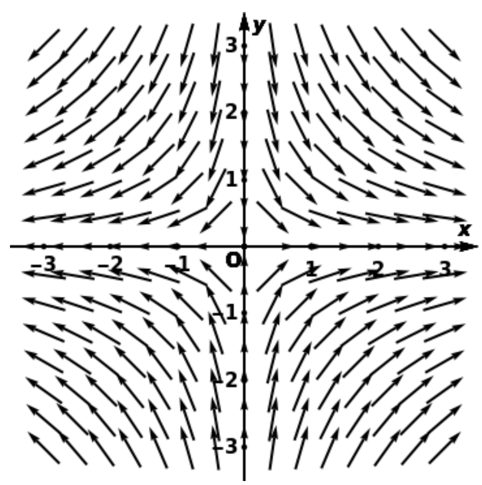
(D)



(E)



(F)



2. (4 points) **No partial credit** for this problem.

Which of the following is the center of mass of the region R bounded by $y = \sqrt{1-x^2}$ and $y = 0$ with density function $\rho(x, y) = x^2 y$? Suppose m is the mass of R.

(A) $(\bar{x}, \bar{y}) = \left(0,\ \dfrac{1}{m}\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1 r^3\cos^2\theta\sin\theta\ \mathrm{d}r\,\mathrm{d}\theta\right)$

(B) $(\bar{x}, \bar{y}) = \left(0,\ \dfrac{1}{m}\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1 r^4\cos^2\theta\sin\theta\ \mathrm{d}r\,\mathrm{d}\theta\right)$

(C) $(\bar{x}, \bar{y}) = \left(0,\ \dfrac{1}{m}\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1 r^5\cos^2\theta\sin^2\theta\ \mathrm{d}r\,\mathrm{d}\theta\right)$

(D) $(\bar{x}, \bar{y}) = \left(0,\ \dfrac{1}{m}\displaystyle\int_0^{\pi}\int_0^1 r^3\cos^2\theta\sin\theta\ \mathrm{d}r\,\mathrm{d}\theta\right)$

(E) $(\bar{x}, \bar{y}) = \left(0,\ \dfrac{1}{m}\displaystyle\int_0^{\pi}\int_0^1 r^4\cos^2\theta\sin^2\theta\ \mathrm{d}r\,\mathrm{d}\theta\right)$

(F) $(\bar{x}, \bar{y}) = \left(0,\ \dfrac{1}{m}\displaystyle\int_0^{\pi}\int_0^1 r^5\cos^2\theta\sin^2\theta\ \mathrm{d}r\,\mathrm{d}\theta\right)$

3. (15 points) Let

$$\vec{\boldsymbol{F}}(x,y) = \langle ye^x,\ e^x + 3y^2 \rangle$$

(a) Is $\vec{\boldsymbol{F}}(x,y)$ conservative? If so, find a potential function. If not, explain why not.

(b) Suppose C is the piecewise smooth curve given by

$$\vec{\boldsymbol{r}}(t) = \begin{cases} \left\langle e^{t^2} - 1,\ e^{t^4} - 1 \right\rangle, & 0 \le t \le 2 \\ \left\langle e^{2t} - 1,\ e^{8t} - 1 \right\rangle, & 2 < t \le 3 \end{cases}$$

Calculate $\int_C \vec{\boldsymbol{F}} \cdot \mathrm{d}\vec{\boldsymbol{r}}$. Justify your answer.

4. (15 points) Evaluate

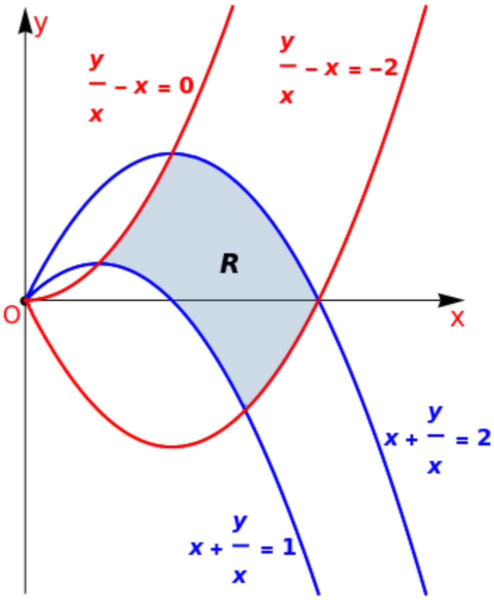
$$\iint_D \cos(x^2 + y^2)\,\mathrm{d}A,$$

where D is the region bounded by the circle $x^2 + y^2 = \dfrac{\pi}{2}$ in the **first** quadrant.

5. (14 points) Let R be the region in the xy-plane (see the graph below) bounded by the curves

$$\frac{y}{x} + x = 1, \quad \frac{y}{x} + x = 2, \quad \frac{y}{x} - x = -2, \quad \text{and } \frac{y}{x} - x = 0.$$

(a) Find a transformation of the form $u = g(x, y)$, $v = h(x, y)$ that maps R onto a rectangle S in the uv-plane. Sketch the rectangle S in the uv-plane.



(b) Use the result in (a) to find an appropriate Jacobian, and use it to evaluate the integral

$$\iint_R \frac{1}{x} \sin\left(\frac{y}{x} + x\right) \,\mathrm{d}A$$

6. (12 points) Evaluate the following triple integral by changing the order of integration.

$$\int_0^{\sqrt{\pi}} \int_z^{\sqrt{\pi}} \int_{x^2}^{\pi} \cos(x^2 + y) \, \mathrm{d}y \, \mathrm{d}x \, \mathrm{d}z$$

Hint: You may have to switch the order of integration at some point.

7. (15 points) Let C be the line segment in $\mathbb{R}^3$ starting from the point $(-1, -1, -2)$ and ending at the point $(2, 3, 0)$.

(a) Compute $\displaystyle\int_C (x^2 + y)\, \mathrm{d}s$

(b) Compute $\displaystyle\int_C \vec{\boldsymbol{F}} \cdot \mathrm{d}\vec{\boldsymbol{r}}$, where $\vec{\boldsymbol{F}} = \langle x - y,\ z,\ -y \rangle$.

8. (5 points) There exists a domain R in the 4th quadrant of area 2 in the xy-plane that we are too lazy to write down. Can you still compute the surface area of the part of the plane $z = 2x - 2y$ above the domain R? If so, do so.

9. (4 points) **No partial credit** for this problem.

Given

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x,$$

find the limits of integration when the order of integration is changed to $\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$.

(A) $\displaystyle\int_0^1 \int_0^{\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x,y,z)\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

(B) $\displaystyle\int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x,y,z)\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

(C) $\displaystyle\int_0^1 \int_0^{\sqrt{x}} \int_{\sqrt{x}}^{1-\sqrt{z}} f(x,y,z)\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

(D) $\displaystyle\int_0^1 \int_{1-\sqrt{x}}^{1} \int_{\sqrt{x}}^{\sqrt{z}} f(x,y,z)\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

(E) $\displaystyle\int_0^1 \int_0^{\sqrt{x}} \int_{1-z}^{\sqrt{x}} f(x,y,z)\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

(F) $\displaystyle\int_0^1 \int_{\sqrt{x}}^{1} \int_{1-z}^{\sqrt{x}} f(x,y,z)\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

10. (4 points) **No partial credit** for this problem.

Use spherical coordinates to express the following sum of integrals as a single integral:

$$\int_{-2\sqrt{2}}^{0}\int_{0}^{\sqrt{8-x^2}}\int_{-\sqrt{8-x^2-y^2}}^{0} x\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x + \int_{0}^{2}\int_{x}^{\sqrt{8-x^2}}\int_{-\sqrt{8-x^2-y^2}}^{0} x\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

(A) $\displaystyle\int_{0}^{\frac{\pi}{4}}\int_{\frac{\pi}{4}}^{\pi}\int_{0}^{2\sqrt{2}} \rho^3\sin^2(\phi)\cos(\theta)\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

(B) $\displaystyle\int_{\frac{\pi}{2}}^{\pi}\int_{\frac{\pi}{4}}^{\pi}\int_{0}^{2\sqrt{2}} \rho^3\sin^2\phi\cos\theta\;\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

(C) $\displaystyle\int_{0}^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\pi}\int_{0}^{2\sqrt{2}} \rho^3\cos^2(\phi)\sin(\theta)\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

(D) $\displaystyle\int_{\frac{\pi}{2}}^{\pi}\int_{\frac{\pi}{4}}^{\pi}\int_{0}^{2\sqrt{2}} \rho^3\cos^2(\phi)\sin(\theta)\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

(E) $\displaystyle\int_{\frac{\pi}{2}}^{\pi}\int_{\frac{\pi}{4}}^{\pi}\int_{0}^{2\sqrt{2}} \rho^3\sin(\phi)\sin^2(\theta)\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

(F) $\displaystyle\int_{\frac{\pi}{2}}^{\pi}\int_{\frac{\pi}{4}}^{\pi}\int_{0}^{2\sqrt{2}} \rho^3\sin^3(\phi)\cos(\theta)\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$