Math 2400, Midterm 1

February 11, 2019

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50 AM
☐	Section 002	Harrison Stalvey	8:00–8:50 AM
☐	Section 003	Daniel Martin	9:00–9:50 AM
☐	Section 004	Albert Bronstein	9:00–9:50 AM
☐	Section 005	Xingzhou Yang	10:00–10:50 AM
☐	Section 006	Mark Pullins	10:00–10:50 AM
☐	Section 007	János Englander	10:00–10:50 AM
☐	Section 008	John Willis	12:00–12:50 PM
☐	Section 009	Taylor Klotz	1:00–1:50 PM
☐	Section 010	János Englander	2:00–2:50 PM
☐	Section 011	Harrison Stalvey	2:00–2:50 PM
☐	Section 012	Xingzhou Yang	3:00–3:50 PM
☐	Section 013	Trevor Jack	4:00–4:50 PM

Question	Points	Score
1	10	
2	4	
3	4	
4	12	
5	12	
6	6	
7	11	
8	10	
9	11	
10	10	
11	10	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 95 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like 100/7 or expressions like ln(3)/2 as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (10 points) **Note: No partial credit for this problem.**

Let $\vec{a} = \langle -3,\, 4,\, 0\rangle$, $\vec{b} = \langle 1,\, -3,\, -1\rangle$. Compute

(a) $|\vec{a}| =$ ______________

(b) $3\vec{a} - 2\vec{b} =$ ______________

(c) The angle between $\vec{a}$ and $\vec{b} =$ ______________

(d) $\vec{a} \times \vec{b} =$ ______________

(e) $\text{proj}_{\vec{a}}\vec{b} =$ ______________

2. (4 points) **Note: No partial credit for this problem.**

The **area** of the triangle with vertices $(a, 0, 0), (0, 2a, 0)$ and $(0, 0, 3a)$ is:

(a) $\dfrac{3a^2}{2}$

(b) $5a^2$

(c) $\dfrac{7a^2}{2}$

(d) $6a^2$

(e) $\dfrac{3a^3}{2}$

3. (4 points) **Note: No partial credit for this problem.**

Let $\vec{a} = \langle -1, 2, 1\rangle$, $\vec{b} = \langle 1, -1, 1\rangle$, and $\vec{c} = \langle -2, -2, 1\rangle$. Compute the volume of the parallelepiped formed by $\vec{a}$, $\vec{b}$, and $\vec{c}$.

(a) 9

(b) 10

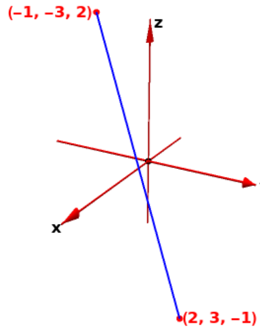
(c) -10

(d) 11

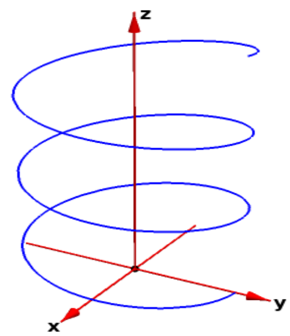
(e) -11

4. (12 points) Match each curve with one of the equations on the right side. Not all equations will be matched.

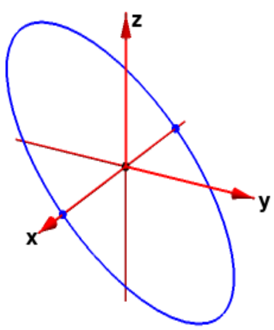
(i) ______



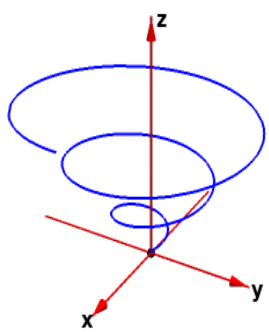
(ii) ______



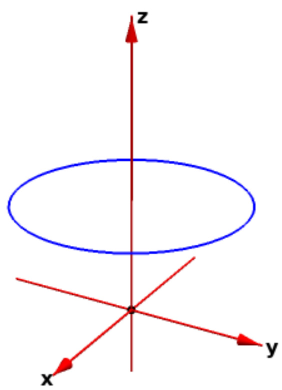
(iii) ______



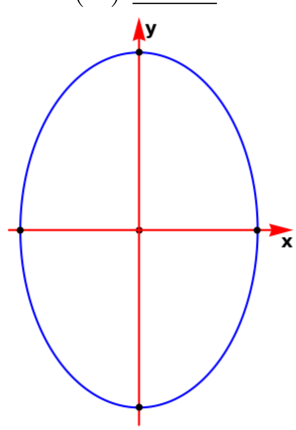
(iv) ______



(v) ______



(vi) ______



(A) $\vec{r}(t) = \langle 3\cos t, 2\sin t\rangle$,
$0 \le t \le 2\pi$

(B) $\vec{r}(t) = \langle 2\sin t, 3\cos t\rangle$,
$0 \le t \le 2\pi$

(C) $\vec{r}(t) = \langle 2 - 3t, 3 - 6t, 3t - 1\rangle$,
$0 \le t \le 1$

(D) $\vec{r}(t) = \langle 2 + 3t, 3 + 6t, 3t - 1\rangle$,
$0 \le t \le 1$

(E) $\vec{r}(t) = \langle \sin t, \cos t, t\rangle$,
$0 \le t \le 6\pi$

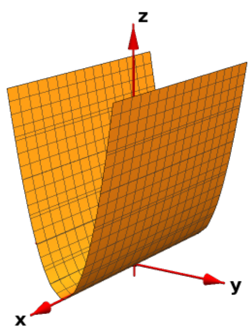
(F) $\vec{r}(t) = \langle \cos t, -\sin t, \sin t\rangle$,
$0 \le t \le 2\pi$

(G) $\vec{r}(t) = \langle t\cos t, t\sin t, t\rangle$,
$0 \le t \le 6\pi$

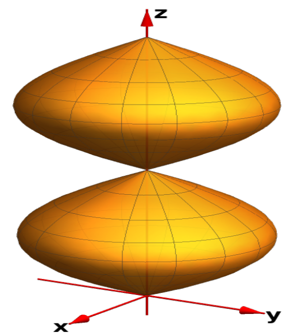
(H) $\vec{r}(t) = \langle \cos t, \sin t, 1\rangle$,
$0 \le t \le 2\pi$

5. (12 points) Match each 3D surface with one of the equations on the right side. Not all equations will be matched.

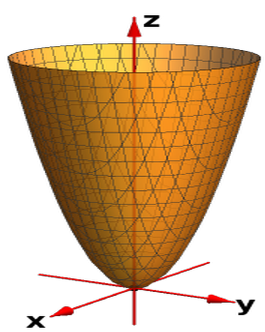
(i) ______



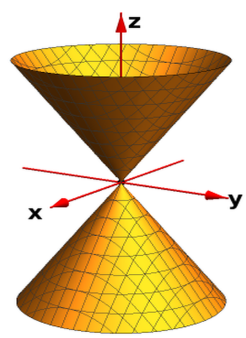
(ii) ______



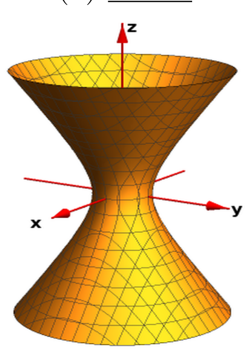
(iii) ______



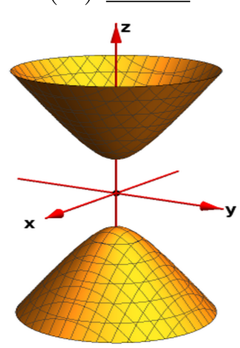
(iv) ______



(v) ______



(vi) ______



(A) $x^2 + y^2 - z = 0$

(B) $x^2 + y^2 - z^2 = 0$

(C) $x^2 + y^2 - z^2 - 1 = 0$

(D) $x^2 + y^2 - z^2 + 1 = 0$

(E) $x^2 + y^2 - \sin^2 z = 0$

(F) $x^2 + y^2 - \cos^2 z = 0$

(G) $z - y^2 = 0$

(H) $z - x^2 = 0$

6. (6 points) Use **spherical coordinates** to describe the solid consisting of points *on and inside* the sphere of radius 3 centered at the origin, but *strictly outside* the sphere of radius 1 centered at the origin, and in the *first* octant.

7. (11 points) Suppose $\vec{r}(t)$ is a differentiable vector function with

$$\vec{r}\,'(t) = \left\langle 2te^{t^2}, \frac{2t}{1+t^2}, \sec^2(t) \right\rangle$$

and $\vec{r}(0) = \langle 0, 0, 0 \rangle$. Find the formula for $\vec{r}(t)$.

8. (10 points) Compute the arc length of the path parameterized by

$$\vec{r}(t) = \left\langle \cos(t), \sin(t), \frac{2}{3}t^{\frac{3}{2}} \right\rangle, \qquad 0 \leq t \leq 3.$$

9. (11 points) Let π be the plane perpendicular to the plane given by the equation $-2x - 2y + z = 8$ and containing the points $(0, 2, 2)$ and $(4, 2, 4)$. Find the equation of π and express it in the form $ax + by + cz + d = 0$.

10. (10 points) Find the distance from the point $(2, -1, 5)$ to the plane $x+y+z+1=0$.

11. (10 points) Find a parametric representation of the surface $z = x^2 + 4y^2$ within the cylinder $x^2 + 4y^2 = 4$. Include the bounds for the parameter(s).