Math 2400, Midterm 2

March 12, 2018

PRINT YOUR NAME: ______________________________

PRINT INSTRUCTOR'S NAME: ______________________________

Mark your section/instructor:

☐	Section 001	Kevin Berg	8:00–8:50
☐	Section 002	Xingzhou Yang	8:00–8:50
☐	Section 003	Albert Bronstein	9:00–9:50
☐	Section 004	Cliff Blakestad	10:00–10:50
☐	Section 005	Albert Bronstein	10:00–10:50
☐	Section 006	Mark Pullins	11:00–11:50
☐	Section 009	Taylor Klotz	11:00–11:50
☐	Section 007	Albert Bronstein	12:00–12:50
☐	Section 008	Martin Walter	1:00–1:50
☐	Section 010	Braden Balentine	2:00–2:50
☐	Section 011	Pedro Berrizbeitia	3:00–3:50
☐	Section 012	Pedro Berrizbeitia	4:00–4:50

Question	Points	Score
1	12	
2	16	
3	13	
4	14	
5	8	
6	14	
7	14	
8	9	
Total:	100	

Honor Code

On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

- No calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated. Use full mathematical or English sentences.
- You have 90 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $\mathbf{100/7}$ or expressions like $\mathbf{\ln(3)/2}$ as is.
- When done, give your exam to your instructor, who will mark your name off on a photo roster.
- We hope you show us your best work!

1. (12 points) Determine whether the following function is continuous at $(0,0)$. Explain why or why not.

$$f(x,y) = \begin{cases} \dfrac{xy^2}{\sqrt{x^4+y^8}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

2. (a) (8 points) Evaluate the double integral $\iint\limits_{R} [x + \sin(y)]\ dA$, where R is a rectangle $R = [0,\ 3] \times [0,\ 3\pi]$.

(b) (8 points) Compute the following double integral by changing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^2 y \cos\left(y^4\right)\ dy\, dx$$

3. (13 points) Find an equation of the tangent plane to the surface given by

$$\vec{r}(u,v) = \langle u+v,\ u-v,\ u^2 - v^3 \rangle$$

at the point $(2, 0, 0)$.

4. Given function $z = f(x, y) = x^3 - 5x + y^2 + 3$,

(a) (7 points) Find the gradient of f at $(-1, 1)$.

(b) (7 points) Find the directional derivative of f at $(-1, 1)$ in the direction of $\vec{v} = \langle 4, -3 \rangle = 4\vec{i} - 3\vec{j}$.

5. (8 points) Let $z = f(x, y) = x^2 + 2y^2 + xy^2$. How many critical points does f have?

A. $\mathbf{0}$

B. $\mathbf{1}$

C. $\mathbf{2}$

D. $\mathbf{3}$

E. $\mathbf{4}$

6. (14 points) Use the **method of Lagrange multipliers** to find the maximum and minimum values of $f(x,y) = x^2 + 2y^2 + xy^2$ subject to the constraint $g(x,y) = 2x + y^2 = 1$.

7. Suppose that $z = F(x, y)$ and that $x = X(u, w)$ and $y = Y(u, w)$, where F, X and Y all have continuous partial derivatives at all points.

Caution: one can view z as a function of x and y, and one can view z as a function of u and w.

Suppose that the following facts are given:

$$\frac{\partial z}{\partial x}(3,4) = q \quad \frac{\partial z}{\partial x}(a,b) = 10 \quad \frac{\partial z}{\partial y}(3,4) = 5 \quad \frac{\partial z}{\partial y}(a,b) = 14$$

$$\frac{\partial x}{\partial u}(a,b) = 10 \quad \frac{\partial x}{\partial w}(a,b) = 8 \quad \frac{\partial y}{\partial u}(a,b) = p \quad \frac{\partial z}{\partial w}(a,b) = 0$$

$$X(p,q) = -2 \quad Y(p,q) = -75 \quad X(a,b) = 3 \quad Y(a,b) = 4$$

(a) (7 points) Use the Chain Rule to find $\dfrac{\partial z}{\partial u}$ when $u = a$ and $w = b$.

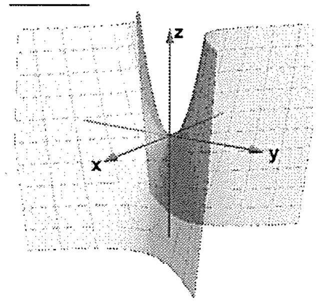
(Your answer may depend on a, b, p, and/or q.)

(b) (7 points) Use the Chain Rule to find $\dfrac{\partial y}{\partial w}$ when $u = a$ and $w = b$.

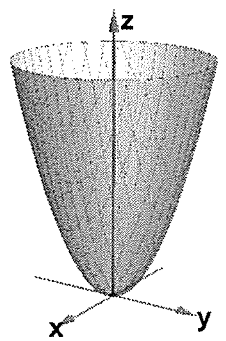
(Your answer may depend on a, b, p, and/or q.)

8. (9 points) Match each 3D surface with one of the contour plots.

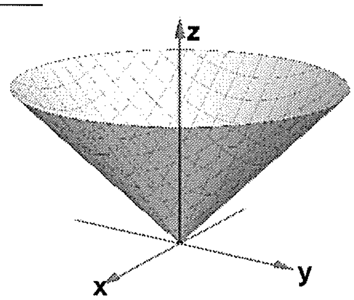
A ______



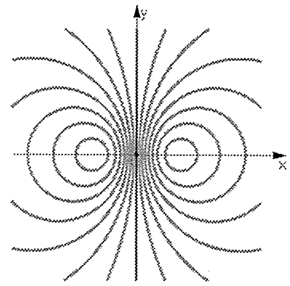
B ______



C ______



(1)



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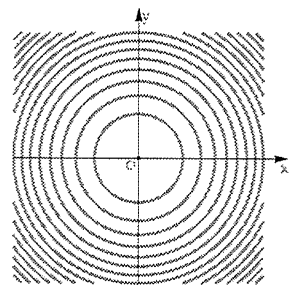
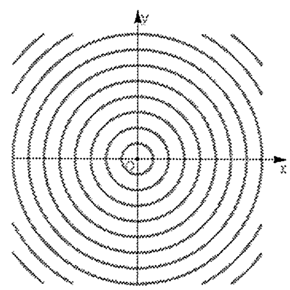
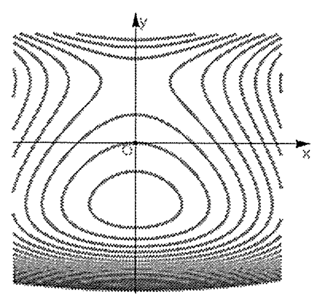


(3)



(4)



(5)