Review Problems

MATH 2400-004, CALCULUS III, FALL 2019

Name:

Instructions for Doing These Problems:

1. When you first attempt the problem, **DO NOT** use any help (books, notes, etc)
2. If you cannot solve the problem during step 1, then you may use books and notes. If you solve it during this step, make a special mark next to the problem number and make sure to review that section.
3. If you cannot solve it during step 2, then I recommend either asking me or a classmate for help. Make sure to mark the problem and read the section.
4. Once you've finished all of the problems, you should focus on practice problems from the corresponding sections of your marked problems. I **highly recommend** doing odd-numbered exercises until you're almost always getting the right answer with no help.

1. Find the domain of $f(x, y) = \sqrt{y}\ln(x - 4)$.

$$\boxed{D = \{(x,y) \mid 0 \leq y,\ 4 < x\}}$$

2. Find the domain of $g(x, y) = \frac{x^2 - xy + y^2}{\sin(\pi xy)}$.

$n \in \mathbb{Z}$

$\sin(\pi xy) \neq 0 \implies xy\pi \neq n\pi \implies xy \neq n$

$$\boxed{D = \{(x,y) \mid xy \notin \mathbb{Z}\}}$$

$\mathbb{Z}$ = Integers

3. Describe the level surfaces of the function $h(x, y) = x^2 - y^2$.

$C = x^2 - y^2$

$\boxed{\text{Hyperbolas}}$

4. Describe the level surfaces of the function $\sigma(x, y) = \ln(x^2 + y^2)$

$$C = \ln(x^2+y^2)$$

$$e^C = x^2+y^2$$

$\boxed{\text{Circles}}$

5. Determine whether or not the following limit exists. If it does, calculate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{x}{x+y}$$

Let $x=0$ $\lim\limits_{y\to 0} \frac{0}{y} = 0$

Let $y=0$ $\lim\limits_{x\to 0} \frac{x}{x} = 1$

$0 \neq 1$

So limit DNE

6. Determine whether or not the following limit exists. If it does, calculate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{3xy}{2x^2+2y^2}$$

Let $x=0$ $\lim\limits_{y\to 0} \frac{0}{2y^2} = 0$

Let $x=y$ $\lim\limits_{y\to 0} \frac{3y^2}{4y^2} = \frac{3}{4}$

$0 \neq \frac{3}{4}$

So limit DNE

7. Determine whether or not the following limit exists. If it does, calculate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$$

Let $y=0$ $\lim\limits_{x\to 0} \frac{0}{x^2} = 0$

Let $x=y^3$ $\lim\limits_{y\to 0} \frac{y^3y^3}{(y^3)^2+y^6} = \lim\limits_{y\to 0} \frac{y^6}{2y^6} = \frac{1}{2}$

$0 \neq \frac{1}{2}$

So limit DNE

8. Determine whether or not the following limit exists. If it does, calculate the limit:

$$\lim_{(x,y)\to(0,0)} \frac{xe^y}{x+y}$$

Let $x=0$ $\lim_{y\to 0} \frac{0}{y} = 0$

$0 \neq 1$

So limit DNE

Let $y=0$ $\lim_{x\to 0} \frac{x}{x} = 1$

9. Find the value c to make this function continuous everywhere. If no c exists, then explain why:

$$f(x,y) = \begin{cases} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} & (x,y) \neq (0,0) \\ c & (x,y) = (0,0) \end{cases}$$

Let $x = r\cos\theta$, $y = r\sin\theta$ $\lim_{r\to 0} \frac{r^2}{\sqrt{r^2+1}-1} \cdot \frac{\sqrt{r^2+1}+1}{\sqrt{r^2+1}+1}$

$= \lim_{r\to 0} \frac{r^2(\sqrt{r^2+1}+1)}{r^2} = \lim_{r\to 0} \sqrt{r^2+1}+1 = 2$

$\boxed{c=2}$

10. Find the value c to make this function continuous everywhere. If no c exists, then explain why:

$$f(x,y) = \begin{cases} \frac{x^2+2y^2}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ c & (x,y) = (0,0) \end{cases}$$

Let $x = r\cos\theta$, $y = r\sin\theta$ $\lim_{r\to 0} \frac{r^2\cos^2\theta + r^2\sin^2\theta + r^2\sin^2\theta}{r}$

$= \lim_{r\to 0} \frac{r^2 + r^2\sin^2\theta}{r} = \lim_{r\to 0} r + r\sin^2\theta = \lim_{r\to 0} r(1+\sin^2\theta)$

$0 \leq 1+\sin^2\theta \leq 2$

$0 \leq r(1+\sin^2\theta) \leq 2r$

By Squeeze Thm, $\lim_{r\to 0} r(1+\sin^2\theta) = 0$ $\boxed{c=0}$

11. Calculate the partial derivatives of $f(x, y) = xy - 3e^y$

$$\frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x - 3e^y$$

12. Calculate the partial derivatives of $g(w, x, y, z) = wx^3 - \cos(xyz) + 3w^2 \ln(yz)$

$$\frac{\partial g}{\partial w} = x^3 + 6w\ln(yz) \qquad \frac{\partial g}{\partial y} = xz\sin(xyz) + \frac{3w^2 z}{yz}$$

$$\frac{\partial g}{\partial x} = 3x^2 w + yz\sin(xyz) \qquad \frac{\partial g}{\partial z} = xy\sin(xyz) + \frac{3w^2 y}{yz}$$

13. Calculate the partial derivatives of $\vec{h}(x, y, z) = \langle x\tan(x) - y, yz + z^4, xyz + 4ye^{xy}\rangle$

$$\frac{\partial h}{\partial x} = \langle \tan(x) + x\sec^2(x), 0, yz + 4y^2 e^{xy}\rangle$$

$$\frac{\partial h}{\partial y} = \langle -1, z, xz + 4e^{xy} + 4xye^{xy}\rangle$$

$$\frac{\partial h}{\partial z} = \langle 0, y + 4z^3, xy\rangle$$

14. Find the equation of the tangent plane to the surface $f(x, y) = \sin(xy) + x$ at the point $(0, 1)$.

$\sin(xy) + x - z = 0$, $\nabla f = \langle y\cos(xy) + 1, x\cos(xy), -1\rangle$

$f(0,1) = 0$ $\quad \nabla f(0,1,0) = \langle 2, 0, -1\rangle$

$$\langle x, y-1, z\rangle \cdot \langle 2, 0, -1\rangle = 0$$

$$\boxed{2x - z = 0}$$

15. Find the equation of the tangent plane to the surface $f(x, y) = xy - x^2 + y$ at the point $(-3, 2)$

$f(-3,2) = -6 - 9 + 2$ $\quad z = xy - x^2 + y$

$= -13$ $\quad xy - x^2 + y - z = 0$

$\nabla f = \langle y - 2x, x + 1, -1\rangle$ $\quad \nabla f(-3, 2, -13) = \langle 8, -2, -1\rangle$

$$\langle x+3, y-2, z+13\rangle \cdot \langle 8, -2, -1\rangle = 0$$

$$\boxed{8(x+3) - 2(y-2) - (z+13) = 0}$$

$\langle 4, 0, 64 \rangle$

16. Find the equation of the tangent plane to the surface $\vec{r}(t,s) = \langle s-t^2, \sin(t), t^3+s^3 \rangle$ when $t = 0$ and $s = 4$.

$\frac{\partial \vec{r}}{\partial t} = \langle -2t, \cos(t), 3t^2 \rangle$ $\quad$ $\frac{\partial \vec{r}}{\partial s} = \langle 1, 0, 3s^2 \rangle$

$\frac{\partial \vec{r}}{\partial t}(0,4) = \langle 0, 1, 0 \rangle$ $\quad$ $\frac{\partial \vec{r}}{\partial s}(0,4) = \langle 1, 0, 48 \rangle$

$\begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 0 & 48 \end{vmatrix} = \langle 48, 0, -1 \rangle$ $\quad$ $\langle 48, 0, -1 \rangle \cdot \langle x-4, y, z-64 \rangle = 0$

$\boxed{48(x-4) - (z-64) = 0}$

17. Find the equation of the tangent plane to the surface $\vec{r}(t,s) = \langle t^3 + 2t^2 - st + 5, s^2t, t-s \rangle$ at the point $(8,0,1)$.

$s^2t = 0$

$s=0$ or $t=0$

If $t=0$, component $1 = 5$, so $s=0$

$t=1$

$\frac{\partial \vec{r}}{\partial t} = \langle 3t^2+4t-s, s^2, 1 \rangle$

$\frac{\partial \vec{r}}{\partial t}(1,0) = \langle 7, 0, 1 \rangle$

$\frac{\partial \vec{r}}{\partial s} = \langle -t, 2st, -1 \rangle$

$\frac{\partial \vec{r}}{\partial s}(1,0) = \langle -1, 0, -1 \rangle$

$\begin{vmatrix} i & j & k \\ 7 & 0 & 1 \\ -1 & 0 & -1 \end{vmatrix} = \langle 0, 6, 0 \rangle$

$\langle x-8, y, z-1 \rangle \cdot \langle 0, 6, 0 \rangle = 0$

$6y = 0$

$\boxed{y = 0}$

18. Use linearization to estimate the value of $f(.9, .9)$ where $f(x,y) = ye^{x-1}$.

$\frac{\partial f}{\partial x} = ye^{x-1}$

$\frac{\partial f}{\partial y} = e^{x-1}$

Easy Point: $(1,1)$

$(1,1,1)$

$\frac{\partial f}{\partial x}(1,1) = 1$

$\frac{\partial f}{\partial y}(1,1) = 1$

$z - 1 = (x-1) + (y-1)$

$z = x + y - 1$

$z(.9, .9) = 1.8 - 1$

$= .8$

$\boxed{f(.9,.9) \approx .8}$

19. Use linearization to estimate the value of $e^{.1}\cos(3)$

$f(x,y) = e^x \cos(y)$ $\quad$ Point: $(.1, 3)$ $\quad$ Easy Point: $(0, \pi, -1)$

$\frac{\partial f}{\partial x} = e^x \cos(y)$

$\frac{\partial f}{\partial y} = -e^x \sin(y)$

$\frac{\partial f}{\partial x}(0,\pi) = -1$

$\frac{\partial f}{\partial y}(0,\pi) = 0$

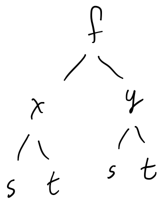
$z + 1 = -1(x-0) + 0(y-\pi)$

$z = -x - 1$

$z = -.1 - 1$

$= -1.1$

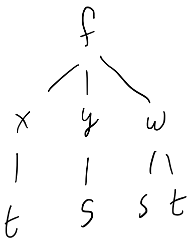
$\boxed{e^{.1}\cos(3) \approx -1.1}$



20. Let $z = f(x, y)$ with x and y as functions of s and t given by $x(s, t) = s^2 - t$ and $y(s, t) = \frac{t}{s}$. Calculate the partials of f with respect to s and t.

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} = \frac{\partial f}{\partial x}(2s) + \frac{\partial f}{\partial y}\left(-t/s^2\right) = 2s\frac{\partial f}{\partial x} - \frac{t}{s^2}\frac{\partial f}{\partial y}$$

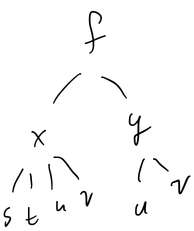
$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} = \frac{\partial f}{\partial x}(-1) + \frac{\partial f}{\partial y}\left(\frac{1}{s}\right) = \frac{1}{s}\frac{\partial f}{\partial y} - \frac{\partial f}{\partial x}$$



21. Let $z = f(x, y, w)$ with x, y, and w as functions of s and t given by $x(s, t) = e^t$ and $y(s, t) = s^3 - s$ and $w(s, t) = s^4 - \sin(ts)$. Calculate the partials of f with respect to s and t.

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial s} = (3s^2 - 1)\frac{\partial f}{\partial y} + (4s^3 - t\cos(st))\frac{\partial f}{\partial w}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial t} = e^t\frac{\partial f}{\partial x} + (-s\cos(ts))\frac{\partial f}{\partial w}$$



22. Let $z = f(x, y)$ with x and y as functions of s, t, u, and v given by $x(s, t, u, v) = uv - e^{st}$ and $y(s, t, u, v) = \cos(u)\cos(v)$. Calculate the partials of f with respect to s, t, u, and v.

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} = (-te^{st})\frac{\partial f}{\partial x}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} = (-se^{st})\frac{\partial f}{\partial x}$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} = v\frac{\partial f}{\partial x} - \sin(u)\cos(v)\frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial v} = u\frac{\partial f}{\partial x} - \sin(v)\cos(u)\frac{\partial f}{\partial y}$$

23. Calculate the directional derivative of $f(x, y) = x^3 + xy - 4$ in the direction of $\vec{u} = \langle 1, 0\rangle$ at the point $(1, 1)$.

$$\nabla f(x,y) = \langle 3x^2 + y, x\rangle \qquad \nabla f(1,1) = \langle 4, 1\rangle$$

$$D_{\vec{u}} f(1,1) = \langle 4, 1\rangle \cdot \langle 1, 0\rangle = \boxed{4}$$

24. Calculate the directional derivative of $f(x, y) = y^2 - x^3 y$ in the direction of $\vec{u} = \langle 2, 1\rangle$ at the point $(-2, 3)$.

$$\nabla f(x,y) = \langle 3x^2 y, 2y - x^3\rangle \qquad \vec{u} = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle$$

$$\nabla f(-2,3) = \langle 36, 14\rangle$$

$$D_{\vec{u}} f(-2,3) = \langle 36, 14\rangle \cdot \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle = \boxed{\frac{72 + 14}{\sqrt{5}}}$$

25. Calculate the maximum rate of change of $f(x,y) = e^x + xy^2$ at the point $(0,3)$.

$\nabla f(x,y) = \langle e^x + y^2, 2xy \rangle$

$\nabla f(0,3) = \langle 10, 0 \rangle$, $|\langle 10,0 \rangle| = \boxed{10}$

26. Determine the direction of the maximum rate of change of $f(x,y) = \sin(xy)$ at the point $(\pi, 2)$.

$\nabla f(x,y) = \langle y\cos(xy), x\cos(xy) \rangle$

$\nabla f(\pi,2) = \langle 2\cos(2\pi), \pi\cos(2\pi) \rangle = \boxed{\langle 2, \pi \rangle}$

27. Determine the equation of the tangent plane to the surface $x^2 - y^2 - 2z^2 = 4$ at the point $(4,2,2)$.

$\nabla f(x,y,z) = \langle 2x, -2y, -4z \rangle$

$\nabla f(4,2,2) = \langle 8, -4, -8 \rangle$

$\boxed{8(x-4) - 4(y-2) - 8(z-2) = 0}$

28. Calculate the equation of the normal line to the surface $x^2 + y^2 + z^2 = 1$ at the point $(1,0,0)$.

$\nabla f(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\nabla f(1,0,0) = \langle 2,0,0 \rangle$

$\boxed{\vec{r}(t) = \langle 1+2t, 0, 0 \rangle}$

29. Find and classify the local extrema of $f(x,y) = x^2 - 3y^2 + 1$

$\frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = -6y$

$(0,0)$ only critical point

$f_{xx} = 2 \quad f_{yy} = -6$

(0,0) is a saddle point

$D = \begin{vmatrix} 2 & 0 \\ 0 & -6 \end{vmatrix} = -12$

30. Find and classify the local extrema of $f(x, y) = x^2y - x - y^2$

$\frac{\partial f}{\partial x} = 2xy - 1$ $\frac{\partial f}{\partial y} = x^2 - 2y$

$x^2 - 2y = 0$

$x^2 = 2y$

$2xy - 1 = 0$

$(2y)x = 1$

$x^3 = 1$

$x = 1 \Rightarrow y = 1/2$

$(1, 1/2)$

$f_{xx} = 2y$ $f_{yy} = -2$

$f_{xy} = f_{yx} = 2x$

$D = \begin{vmatrix} 2y & 2x \\ 2x & -2 \end{vmatrix} = -4y - 4x$

$(1, 1/2)$ is a saddle point

31. Find and classify the local extrema of $f(x, y) = x^4 + y^2x - xy$

$\frac{\partial f}{\partial x} = 4x^3 + y^2 - y$ $\frac{\partial f}{\partial y} = 2yx - x$

$2yx - x = 0$

$x(2y - 1) = 0$

$x = 0$ or $y = 1/2$

$x = 0$

$y^2 - y = 0$

$y(y-1) = 0$

$(0,0), (0,1)$

$y = 1/2$

$4x^3 + \frac{1}{4} - \frac{1}{2} = 0$

$4x^3 - 1/4 = 0$

$x^3 = \frac{1}{16}$ $\left(\frac{1}{\sqrt[3]{16}}, \frac{1}{2}\right)$

$x = \sqrt[3]{1/16}$

$f_{xx} = 12x^2$ $f_{yy} = 2x$

$f_{xy} = f_{yx} = 2y - 1$

$D = \begin{vmatrix} 12x^2 & 2y-1 \\ 2y-1 & 2x \end{vmatrix} = 24x^3 - (2y-1)^2$

$(0,0)$: $D = 0 - 1 = -1$

Saddle point

$(0,1)$: $D = 0 - (2-1)^2$

< 0

Saddle point

$\left(\frac{1}{\sqrt[3]{16}}, \frac{1}{2}\right)$

$D = \frac{24}{16} - 0$

$12\left(\frac{1}{\sqrt[3]{16}}\right)^2 > 0$

Local Min

32. Find and classify the absolute extrema of $f(x, y) = x^2 + 2y^2 - 3$ on the disk $D = \{(x, y) \mid x^2 + y^2 \leq 3\}$.

$f_x = 2x$ $f_y = 4y$

$(0,0)$

$f_{xx} = 2$

$D = \begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = 8$

$f(0,0) = -3$

$3 + y^2 - 3$

$= y^2$

$f(x,y) = y^2$ on boundary

$f(y) = y^2$

Max is 3 on $x^2 + y^2 = 3$

Min is 0

Max $= 3$

Min $= -3$

$x^2 + y^2 = 3$

$x^2 + 2y^2 - 3$

$= x^2 + y^2 + y^2 - 3$

33. Calculate the maximum and minimum values attained by $f(x, y) = xy^2$ under the constraint $x^2 + y^2 = 3$

$y^2 = \lambda 2x$

$2xy = \lambda 2y$

Sps $y = 0$.

$\Rightarrow \lambda = 0$ or $x = 0$

Since $x^2 + y^2 = 3$, then $x \neq 0$, so

$\lambda = 0$, $x = \pm\sqrt{3}$

$(\sqrt{3}, 0)$

$(-\sqrt{3}, 0)$

Sps $y \neq 0$

$2x = \lambda 2$

$x = \lambda$

$\Rightarrow y^2 = 2x^2$

$x^2 + 2x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$y = \pm\sqrt{2}$

$(1, \sqrt{2})$

$(-1, -\sqrt{2})$

$(1, -\sqrt{2})$

$(-1, \sqrt{2})$

$f(\sqrt{3}, 0) = f(-\sqrt{3}, 0) = 0$

$f(1, \sqrt{2}) = f(1, -\sqrt{2}) = \boxed{2 \leftarrow \text{Max}}$

$f(1, -\sqrt{2}) = f(-1, \sqrt{2}) = \boxed{-2 \leftarrow \text{Min}}$

34. Calculate the maximum and minimum values attained by $f(x, y, z) = xyz$ under the constraint $x^2 + y^2 + z^2 = 1$.

$yz = \lambda 2x$

$xz = \lambda 2y$

$xy = \lambda 2z$

If $x = 0$

$y = 0$ or $z = 0$

& $\lambda = 0$

Since $x^2 + y^2 + z^2 = 1$

If $y = 0$, $z \neq 0$, then

$z = \pm 1$

Same conclusion for letting any variable $= 0$, so we get

$(\pm 1, 0, 0)$

$(0, \pm 1, 0)$

$(0, 0, \pm 1)$

$x, y, z \neq 0$

$\frac{yz}{2x} = \frac{xz}{2y} \Rightarrow 2z(y^2 - x^2) = 0$

$\frac{xz}{2y} = \frac{xy}{2z} \Rightarrow 2x(z^2 - y^2) = 0$

So $x^2 = y^2 = z^2$

$3x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{3}}$

$\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$

Max $= \left(\frac{1}{\sqrt{3}}\right)^3$ Min $= \left(-\frac{1}{\sqrt{3}}\right)^3$

35. Calculate two points on the circle $x^2 + y^2 = 4$: the point furthest from $(3, 3)$ and the point closest to $(3, 3)$.

$f(x, y) = (x-3)^2 + (y-3)^2$

$2(x-3) = \lambda 2x$

$2(y-3) = \lambda 2y$

$2x(1-\lambda) = 6$

$2y(1-\lambda) = 6$

$2x(1-\lambda) = 2y(1-\lambda)$

What if $\lambda = 1$?

Then $0 = 6$

Crazy!

So $\lambda \neq 1$ &

$2x = 2y$

$x = y$

$2x^2 = 4$

$x = \pm\sqrt{2}$

$(\sqrt{2}, \sqrt{2})$ closest

$(-\sqrt{2}, -\sqrt{2})$ furthest

36. Describe the object whose volume can be interpreted as the double integral

$$\iint_R 10\,dA$$

where $R = [0,1] \times [-5,2]$.

It's a box of dimension $1 \times 7 \times 10$

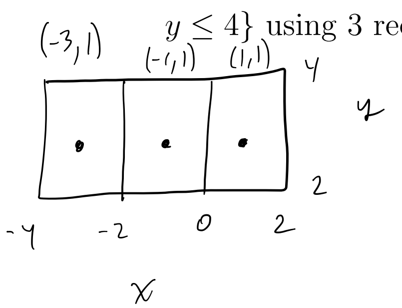
37. True/False: The integral

$$\iint_R \sqrt{1-x^2}\,dA$$

where $R = [-1,1] \times [-1,1]$, represents the volume of the upper half-sphere of radius 1.

False. The base wouldn't be a rectangle for a half-sphere

38. Estimate the double integral $\iint_R x^2 - y^2\,dA$ where $R = \{(x,y) \mid -4 \le x \le 2,\ 2 \le y \le 4\}$ using 3 rectangles of equal area.



$\Delta A = 4$

$V \approx 4\left(f(-3,1) + f(-1,1) + f(1,1)\right)$

$= 4(8 + 0 + 0)$

$= \boxed{32}$

39. Evaluate the double integral $\iint_R x^4(1+y)^3\,dA$ where $R = [0,3] \times [0,5]$.

$\int_0^5 \int_0^3 x^4 (1+y)^3\,dx\,dy$

$= \int_0^5 \frac{x^5}{5}(1+y)^3 \Big|_{x=0}^{x=3} dy$

$= \frac{243}{5} \int_0^5 (1+y)^3\,dy$

$= \frac{243}{5}\left(\frac{(1+y)^4}{4} \Big|_{y=0}^{y=5} \right)$

$= \boxed{\left(\frac{243}{5}\right)\left(\frac{6^4}{4}\right) - \left(\frac{243}{5}\right)\left(\frac{1}{4}\right)}$

40. Evaluate the double integral $\iint_R xe^{xy}\,dA$ where $R = [-3,1] \times [7,9]$.

$$\int_{-3}^{1}\int_{7}^{9} x e^{xy}\,dy\,dx \qquad u = xy \quad du = x\,dy$$

$$= \int_{-3}^{1}\int_{7x}^{9x} e^{u}\,du\,dx$$

$$\int_{-3}^{1} e^{9x} - e^{7x}\,dx$$

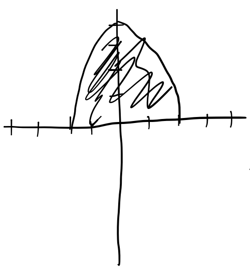
$$= \frac{e^{9x}}{9} - \frac{e^{7x}}{7}\Big|_{x=-3}^{x=1} = \boxed{\frac{e^{9}}{9} - \frac{e^{7}}{7} - \frac{e^{-27}}{9} + \frac{e^{-21}}{7}}$$

41. Evaluate the double integral $\iint_R \ln(x)y\,dA$ where $R = [4,5] \times [-1,1]$.

$$\int_{4}^{5}\int_{-1}^{1} \ln(x)y\,dy\,dx$$

$$= \int_{4}^{5} \ln(x)\frac{y^2}{2}\Big|_{y=-1}^{y=1} dx = \boxed{0}$$

42. Evaluate the double integral $\iint_D 5x^2y\,dA$ where D is the region bounded by $y = 4 - x^2$ and the x-axis.



$$\int_{-2}^{2}\int_{0}^{4-x^2} 5x^2y\,dy\,dx$$

$$= \int_{-2}^{2} \frac{5x^2y^2}{2}\Big|_{y=0}^{y=4-x^2} dx$$

$$= \int_{-2}^{2} \frac{5x^2(16-8x^2+x^4)}{2}\,dx$$

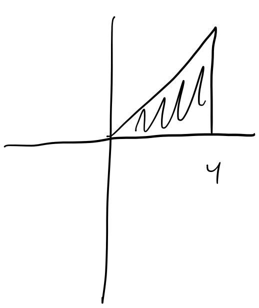
$$= \frac{5}{2}\int_{-2}^{2} 16x^2 - 8x^4 + x^6\,dx$$

$$= 5\int_{0}^{2} 16x^2 - 8x^4 + x^6\,dx$$

$$= 5\left(\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{x^7}{7}\right)\Big|_{x=0}^{x=2}$$

$$= \boxed{5\left(\frac{16}{3}\cdot 8 - \frac{8}{5}\cdot 32 + \frac{128}{7}\right)}$$

43. Evaluate the double integral $\iint_D \cos(x^2)\,dA$ where D is the region bounded by $y = x$, $x = 4$ and the x-axis.



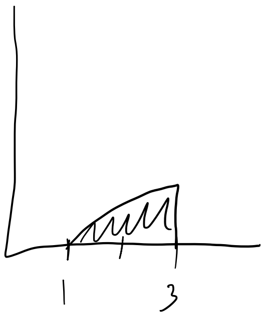
$$\int_{0}^{4}\int_{0}^{x} \cos(x^2)\,dy\,dx$$

$$= \int_{0}^{4} x\cos(x^2)\,dx$$

$$= \frac{\sin(x^2)}{2}\Big|_{x=0}^{x=4}$$

$$= \boxed{\frac{\sin(16)}{2}}$$

44. Evaluate the double integral $\iint_D xe^y \, dA$ where D is the region bounded by $y = \ln(x)$, $x = 3$, and the x-axis.



$$\int_1^3 \int_0^{\ln(x)} x e^y \, dy \, dx$$

$$= \int_1^3 x e^y \Big|_{y=0}^{y=\ln(x)} dx$$

$$= \int_1^3 x^2 - x \, dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} \Big|_{x=1}^{x=3} = \boxed{\left(9 - \frac{9}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)}$$

45. Evaluate the double integral $\iint_D \frac{y}{1+x^2} \, dA$ where D is the region bounded by $x = -1$, $x = 1$, $y = 0$ and $y = 8$.

$$\int_{-1}^1 \int_0^8 \frac{y}{1+x^2} \, dy \, dx$$

$$= \frac{1}{2} \int_{-1}^1 \frac{y^2}{1+x^2} \Big|_{y=0}^{y=8} dx$$

$$= 32 \int_{-1}^1 \frac{1}{1+x^2} \, dx$$

$$= 32 \left(\arctan(x)\right) \Big|_{x=-1}^{x=1}$$

$$= 32 \left(\frac{\pi}{2}\right) = 16\pi$$