1. The sides of a square are all increasing uniformly at a rate of 3 inches/minute. At what rate is the area of the square increasing when the side length is 10 inches?

Let $x$ be the side length of the square. We have $A = x^2$ and thus

$$\frac{dA}{dt} = 2x \frac{dx}{dt}.$$ 

We plug in to get

$$\frac{dA}{dt} = 2(10 \text{ inches}) \left( 3 \frac{\text{inches}}{\text{minute}} \right) = 60 \text{ in}^2/\text{min}.$$ 

2. A particle moves along the graph of $y = \tan(x)$. Its velocity in the $x$-direction ($dx/dt$) is 5 units per minute. When $x = \frac{\pi}{4}$, what is its velocity in the $y$-direction ($dy/dt$)?

$y = \tan(x)$ and so $\frac{dy}{dt} = \sec^2(x) \frac{dx}{dt}$

$$\frac{dy}{dt} = \sec^2 \left( \frac{\pi}{4} \right) \cdot (5 \text{ units/min}) = \left( \sqrt{2} \right)^2 \cdot (5 \text{ units/min}) = 10 \text{ units/min}.$$
3. A car is traveling north toward an intersection at a rate of 60 mph while a truck is traveling east away from the intersection at a rate of 50 mph. Find the rate of change of the distance between the car and truck when the car is 3 miles south of the intersection and the truck is 4 miles east of the intersection.

\[
\frac{dx}{dt} = 50 \text{ miles/hr} \quad \quad \frac{dy}{dt} = -60 \text{ miles/hr}
\]

\[
x^2 + y^2 = z^2
\]

When \( x = 4 \) and \( y = 3 \), we have \( z = \sqrt{9 + 16} = 5 \) miles.

Differentiating the equation above gives

\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}
\]

\[
\frac{dz}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{z} = \frac{(4 \text{ mi})(50 \text{ mi/hr}) + (3 \text{ mi})(-60 \text{ mi/hr})}{5 \text{ mi}} = 4 \text{ miles/hr}.
\]

The distance between them is increasing at 4 miles/hr.
4. A rectangle of length $\ell$ and width $w$ has a constant area of 1200 in$^2$. The side lengths are changing while keeping the area the same. Suppose that at a particular instant the length is increasing at 6 in/min and the width is decreasing at 2 in/min.

(a) Find the dimensions of the rectangle at this instant.

\[ 1200 = \ell w \]
\[ 0 = \frac{d\ell}{dt} w + \ell \frac{dw}{dt} \]
\[ \frac{d\ell}{dt} = 6 \text{ in/min} \]
\[ \frac{dw}{dt} = -2 \text{ in/min} \]
\[ \frac{1200}{w} = \ell \]
\[ 0 = 6w - 2\ell \]
\[ 0 = 6w - 2(1200/w) \]
\[ 6w^2 = 2400 \]
\[ w^2 = 400 \]
\[ w = 20 \text{ inches} \]
\[ \ell = 1200/20 = 60 \text{ inches} \]

(b) At this same instant, is the length of the diagonal increasing or decreasing? At what rate?

\[ D^2 = \ell^2 + w^2 \]
\[ 2D \frac{dD}{dt} = 2\ell \frac{d\ell}{dt} + 2w \frac{dw}{dt} \]
\[ \frac{dD}{dt} = \frac{\ell \frac{d\ell}{dt} + w \frac{dw}{dt}}{D} \]

When $\ell = 60$ and $w = 20$, $D = \sqrt{60^2 + 20^2} = \sqrt{3600 + 400} = \sqrt{4000} = 20\sqrt{10}$ inches.

\[ \frac{dD}{dt} = \frac{(60 \text{ in})(6 \text{ in/min}) + (20 \text{ in})(-2 \text{ in/min})}{20\sqrt{10} \text{ in}} = \frac{320}{20\sqrt{10}} \text{ in/min} \approx 5.06 \text{ in/min}. \]

The length of the diagonal is increasing at approximately 5.06 inches per minute.
5. An FBI agent with a powerful spyglass is located in a boat anchored 0.4 km offshore. A gangster under surveillance is walking along the shore. Assuming the shoreline is straight and that the gangster is walking at the rate of 2 km/hr, how fast must the FBI agent rotate the spyglass to track the gangster when the gangster is 1 km from the point on the shore nearest to the boat? (In other words, find \( \frac{d\theta}{dt} \).)

\[
\tan \theta = \frac{x}{0.4} \quad \text{sec}^2 \theta \frac{d\theta}{dt} = \frac{1}{0.4} \frac{dx}{dt} = 2.5 \frac{dx}{dt}
\]

\[
\frac{dx}{dt} = 2 \text{ km/hr}
\]

When \( x = 1 \), we have \( \tan \theta = 1/0.4 \) and \( \theta = \arctan(1/0.4) = \arctan(2.5) \)

Useful trig identity: \( 1 + \tan^2 * = \sec^2 * \) for any value of *.

\[
\frac{d\theta}{dt} = \frac{2.5 \frac{dx}{dt}}{\sec^2 \theta} = \frac{2.5(2 \text{ km/hr})}{\sec^2(\arctan(2.5))} = \frac{5}{1 + \tan^2(\arctan(2.5))} = \frac{5}{1 + (2.5)^2} = \frac{5}{7.25} = \frac{20}{29} \text{ radians/hr.}
\]