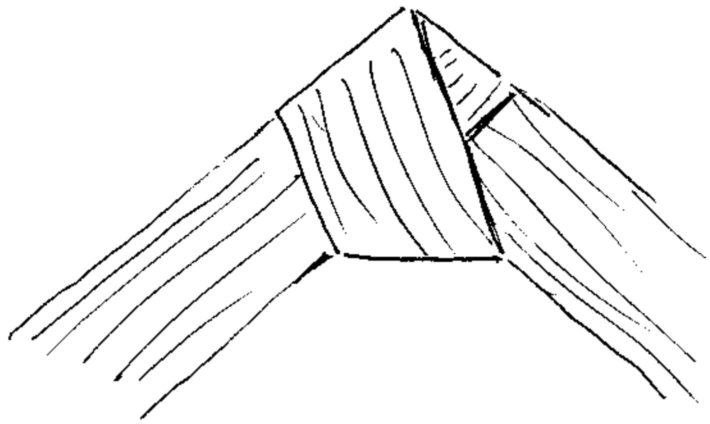
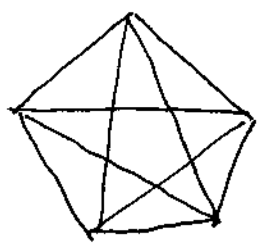


Take this configuration.

1. In this diagram



there are four pairs of parallel lines. (The two horizontal lines are not known yet to be parallel. See step 9 below.)

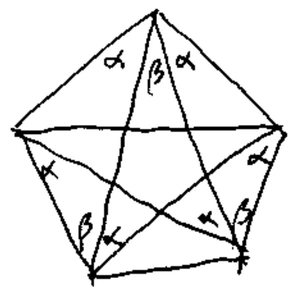
2. When a strip is folded, the two obtuse anges formed are ~~for~~ congruent.

3. Therefore the top three angles of the pentagon are congruent; likewise the bottom two angles.

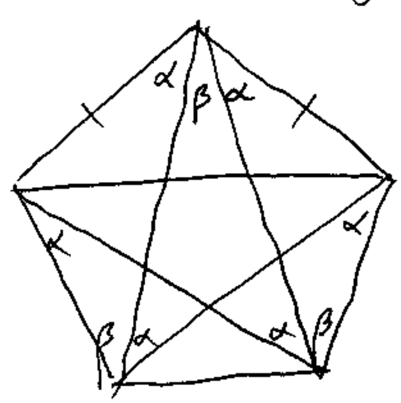
4. Consider these three angles at the top

α is the acute angle of one fold; δ
is the " " " another fold.
These are congruent folds, since they coincide at the top. Therefore $\alpha = \delta$.
Thus, at the top we have

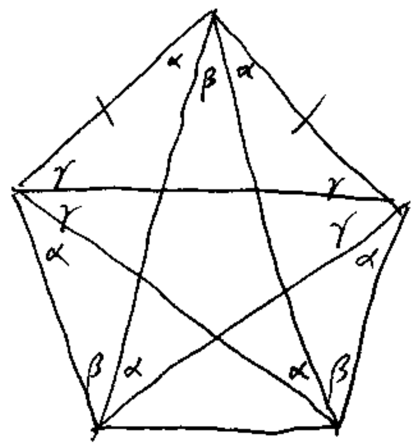
5. By alternate interior angles, using ~~#~~ 1,



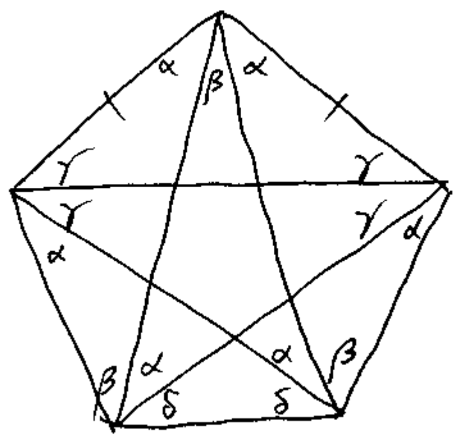
6. the two folds at the top are congruent, hence the two top segments are congruent



7. By isoceles triangle (at top), and further applications of alternate interior angles, we have

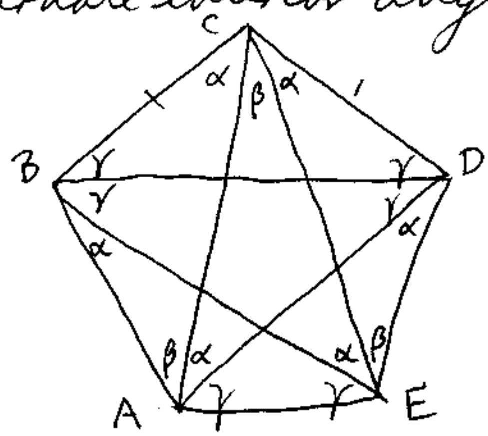


8. By step 3, we have



9. We now have enough congruent angles to assert that the two horizontal lines are parallel

10. By alternate interior angles



11. By ASA, $\triangle BDE \cong \triangle DBA$. Thus $BA \cong DE$.

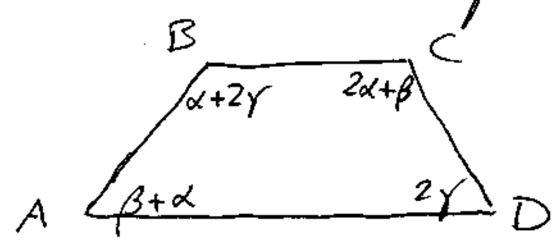
12. By step 3, $2\gamma = \alpha + \beta$.

13. By considering the fold at C, we have

$$\alpha + 2(\alpha + \beta) = 180^\circ$$

or $$3\alpha + 2\beta = 180^\circ.$$

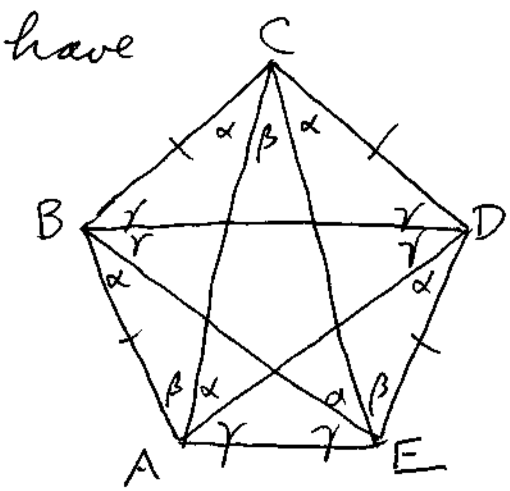
14. Consider the quadrilateral ABCD



By step 1, two sides are parallel. By step 12, $\angle A \cong \angle D$ and $\angle B \cong \angle C$. One easily verifies, for such a quadrilateral, that $BA \cong CD$.

Thus $AB \cong BC \cong CD \simeq DE$. (By 6 & 11)

15. We now have



16. Now $\triangle ABC$ is isosceles, so $\beta = \alpha$

17. From steps 13 and 16,

$$5\alpha = 180^\circ$$
$$\alpha = 36^\circ = \beta.$$

18. From steps 12 and 17

$$\gamma = \tfrac{1}{2}(\alpha+\beta) = \tfrac{1}{2}(\alpha+\alpha) = \alpha = 36^\circ$$

19. Thus the obtuse angles at A, B, C, D, E are each 108°

20. $\triangle CBA \cong \triangle BAE$, so $AE \cong BA$. In other words, all five sides are congruent. (Using Step 14.)

21. From Steps 19 & 20, we have a regular pentagon.