

My research interests are mathematical modeling and solving systems of nonlinear reaction-diffusion equations. This work in applied mathematics utilizes my training in mathematical physics, differential equations, asymptotical and perturbation methods, and numerical solutions of partial differential equations. These areas have been the building blocks for my research in the mathematical modeling of frontal polymerization. My theoretical study has so far focused on the propagation of the polymerization waves and their velocities, the spatial profiles of the species involved, the degree of conversion of monomer, and the temperature of the mixture. This area of research opens the door to fascinating studies at the interface between mathematics, physics, chemistry, and biology. One such multi-disciplinary topic is combustion theory (exothermic polymerization waves are in many respects similar to combustion waves) and another involves the emerging field of tissue engineering and tissue growth. This later application typically deals with diffusion-reaction equations, which can, in some cases lead to the propagation of degradation fronts as discussed later in this statement.

Dissertation Research

Frontal polymerization (FP) is a relatively new area of polymer science with potential for engineering applications. Unlike traditional polymerization processes, polymers produced by this means tend to have a superior properties and yet with simpler and less expensive production equipment. Frontal polymerization is a method of converting monomer into polymer by means of a self-propagating reaction wave. In a typical FP experiment, monomer and initiator are placed into a test tube. Upon initiation, a thin reaction zone travels through the mixture leaving polymer product in its wake (Fig. 1a). There are two modes of FP, isothermal and exothermal, each, for example, giving a different front velocity. By correctly modeling such modes into mathematical equations, one can control the parameters as needed to dictate the velocity. During my dissertation research, I have modeled the FP experiment by a system of nonlinear reaction-diffusion equations describing the amount of reactants and the temperature of the system at some temporal and spatial point along the length of a test tube. My strategy has been to study two main modes of FP (isothermal and exothermal) independently, and then incorporate them into a general model that accounts for both modes.

Isothermal FP is a critical process behind the fabrication of polymer product for optical devices since it can generate polymers having an index of refraction that is non-uniform in space (gradient index). In this context, I studied free-radical polymerization waves that propagate by means of an isothermal front that exhibits strong gel effect (a phenomenon where an increased viscosity of the polymer region results in an acceleration in the overall rate of polymerization and hence an increase in the conversion of monomers). After validating my numerical model against analytical solution, I have shown that the model not only captured the strong gel effect successfully, but it also enabled the quantification of important features such as the propagation velocity of the front.

Exothermic polymerization waves have been particularly interesting to chemists, as they have reduced sensitivity to substances that prevent polymerization and reduce the front velocity. To better understand the mechanisms involved, I derived a model for the copolymerization of thiol-ene, where thiol and ene are each different monomers. The model tracked the evolution of the species and the temperature balance in the system. Both approximate analytical solutions and full numerical solutions of the problem were found, and compared with experimental results when possible. The model was particularly successful at capturing intriguing phenomena. For instance, I showed that at sufficiently low initial concentrations of either monomer, the propagating front is plagued by bulk polymerization, and at even lower concentrations, the front fails to form (Fig. 1b). The model also predicted the existence of an oscillatory propagation of the polymerization wave this was the consequence of an instability of a uniformly propagating wave.

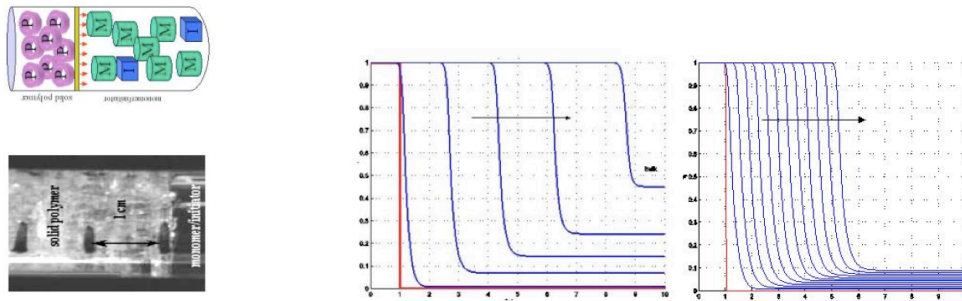


Figure 1: (a) experimental photo and corresponding schematic of monomer/initiator system and solid/gel polymer separated by reaction front.
 (b) degree of conversion of monomer as a function of space, at progressive snapshots of time; first plauged by bulk polymerization and then alleviated by inhibition.

A final contribution of my model was to describe systems in which both isothermal and exothermic free-radical polymerization waves occur simultaneously. Consistent with experimental setup, I considered a polymer test tube, containing its monomer-initiator mixture, placed in a hot thermostatic bath. An isothermal front propagates from the wall of the tube towards the center, and an exothermic front propagates from the center of the tube moving outward towards the wall. The model enabled a relationship between the parameters governing the state of the exothermic and isothermal fronts. A major result of my analysis was to predict that decreasing the strength of gel effect and the initial monomer concentration both lead to a substantial outwardly propagating exothermic front that overpowers the inwardly propagating isothermal front. These effects were further strengthened by increasing the initial initiator concentration and the tube radius. For sufficiently large radii, however, the isothermal front changed regimes to an exothermic one as it continues to propagate towards the center.

In summary, although frontal polymerization is a complex phenomenon, mathematical studies are critical to provide quantitative insights to chemists and chemical engineers, which has motivated me to continue working in this field. In addition, the expected benefits of FP over traditional polymerization techniques have been attractive to industrial manufacturers for the following reasons: polymer production is both rapid and uniform, it has a lower energy cost, reduced waste production (environmentally friendly polymers), and increased control of product features and quality.

Current Research Interest and Future Directions

My main interest of expansion is biological mathematics; another application of reaction-diffusion equations is in *ex vivo* tissue regeneration. The principle behind tissue engineering is to inject damaged tissue with a population of cells that can stimulate repair mechanisms and restore normal function. These cells often need a mechanical support, referred to as scaffold, which can resist physiological loads and ensure proper tissue development.

The growth of new tissue in a scaffold depends on a delicate balance between the cell's matrix production and the kinetics of scaffold degradation. Understanding this balance has been critical for the design of bio-degradable gels that can replace damaged cartilage *in vivo*. Research progresses in this area have been hindered by the following issue: if enzymatic-mediated gel degradation occurs too fast, the scaffold dissolves and cells are unable to produce a collagen matrix in time; if too slow, matrix production is obstructed and cells perish. Biodegradable hydrogel scaffold must therefore possess degradation kinetics that are finely tuned in terms of chondrocyte enzyme and matrix production (Fig. 2). Since trial and error experimental approaches have so far failed in designing such a gel, my plan is to approach the problem from a mathematical modeling side. In particular, the literature shows that it is possible to describe the hydrogel as a continuous and permeable solid that can degrade in time through the dissolution of cross-links. The presence of cells can then be described as spherical inclusions that can produce and diffuse two types of molecules through the gel: (a) an enzyme that can cleave hydrogel cross-links and (b) extra-cellular matrix molecules that can deposit and make new tissue. Interestingly, literature has shown that this approach leads to a **reaction-diffusion model that shares many common features with the physics of frontal polymerization** that I previously worked on. Indeed, when cells are able to locally degrade the gel via the production of an enzyme, the dynamic of the system strongly depends on the competition between diffusion and degradation kinetics. Thus, when the system is diffusion limited, it exhibits a propagating degrading front around the cells that is likely to play a strong role in tissue development.

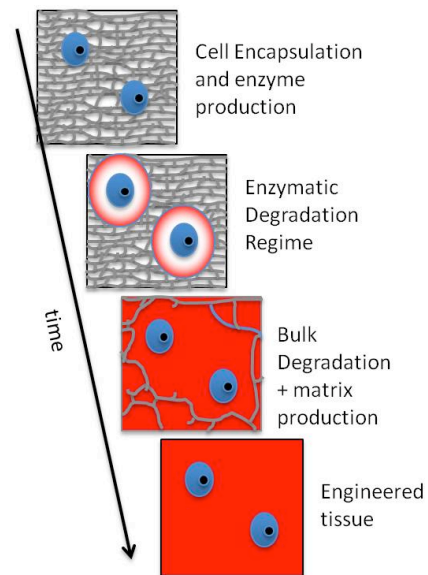


Figure 2: Schematic of scaffold degradation and growth of tissue around cells. Propagating degradation front can be observed around cells (second figure)

Very little is known about what governs the velocity of the front and how it can be tuned by scaffold design. I strongly believe that my background and experience will enable me to positively contribute to the field. Furthermore, applications of such research through the development of mathematical models is likely to open the door to the computational design of patient-specific artificial tissues.