Math 8174: Homework 4

Due March 4–6, 2009

1. Let $D_{2r} = G(r, r, 2)$ be the dihedral group of order $2r$.

   (a) Show that D_{2r} is isomorphic to
   $$\langle s_1, s_2 \mid s_1^2 = s_2^2 = 1, (s_1 s_2)^r = 1 \rangle.$$

   (b) Completely classify the irreducible D_{2r}-modules.
   Hints: All irreducible D_{2r}-modules have dimension ≤ 2. Treat r even and r odd separately.

   (c) Find the characters of the irreducible D_{2r}-modules.

2. Let V be the natural module of S_n (see Homework 3, Problem 4).

   (a) Compute the character $\chi_V : S_n \to \mathbb{C}$ of V.

   (b) Deduce the character $\chi^{(n-1,1)}$ of the irreducible S_n-module $S^{(n-1,1)}$.

3. Show that every S_n-module gives a natural irreducible $G(r, 1, n)$-module (construct the corresponding module). Show that not all irreducible $G(r, 1, n)$-modules are obtained in this way.

4. Let (P_w, Q_w) be the pair of tableaux obtained from $w \in S_n$ by the RSK-correspondence.

   (a) Find w, when the shape of P_w is (1^n) and (n),

   (b) What does the number of rows of $\mathrm{sh}(P_w)$ say about the permutation w?

   (c) What does the length of the first row of $\mathrm{sh}(P_w)$ say about the permutation w?