Math 4140: Homework 5

Due February 18, 2009

1. Suppose s_u and s_v are reflections in the hyperplanes H_u and H_v, respectively. Show that if H_u and H_v are orthogonal, then s_u and s_v commute. Is this true for any other angle between H_u and H_v?

2. Let $V = \mathbb{R}^3 = \mathbb{R}\text{-span}\{e_1, e_2, e_3\}$, and let $(\cdot, \cdot) : V \times V \to \mathbb{R}$ be the usual inner product. For each of the following collection of hyperplanes, identify the angles between the hyperplanes, and the reflection group the reflections in these hyperplanes generate (up to isomorphism).

   (a) H_{e_1}, H_{e_2}, and H_{e_3}.

   (b) $H_{e_1-e_2}$, $H_{e_3-e_2}$, and $H_{e_1+e_2+e_3}$.

   (c) $H_{e_2-e_1}$, $H_{e_2-e_3}$, and $H_{e_1+e_2}$.

3. Let

$$E = \left\{ a_1e_1 + a_2e_2 + \cdots + a_ne_n \in \mathbb{R}^n \;\middle|\; a_1 + a_2 + \cdots + a_n = 0, a_1, a_2, \ldots, a_n \in \mathbb{R} \right\}.$$

   (a) Show that E is a subspace of $\mathbb{R}^n$,

   (b) Find the dimension of E,

   (c) Show that

$$R = \{e_i - e_j \mid 1 \leq i, j \leq n\}$$

   is a root system of E (be sure to check all the axioms).