Math 4140: Homework 11

Due April 28, 2009

1. Let

$$\mathfrak{sp}_4(\mathbb{C}) = \mathfrak{gl}_4(J, \mathbb{C}), \qquad \text{where} \quad J = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}.$$

   (a) Find a natural basis for $\mathfrak{sp}_4(\mathbb{C})$.

   (b) Find two different copies of $\mathfrak{sl}_2(\mathbb{C})$ inside of $\mathfrak{sp}_4(\mathbb{C})$.

   (c) For each copy of $\mathfrak{sl}_2(\mathbb{C})$,

      i. Decompose $\mathfrak{sp}_4(\mathbb{C})$ as an $\mathfrak{h}$-module where $\mathfrak{h} \subseteq \mathfrak{sl}_2(\mathbb{C})$. That is, identify the eigenspaces and their eigenvalues.

      ii. Describe the irreducible $\mathfrak{sl}_2(\mathbb{C})$-submodules of $\mathfrak{sp}_4(\mathbb{C})$.

   (d) Show how the irreducible $\mathfrak{sl}_2(\mathbb{C})$-modules fit together into a set of roots, and give the corresponding Dynkin diagram.