Math 4140: Homework 1

Due January 21, 2009

1. Let
$$\mathrm{O}_n(\mathbb{C}) = \{g \in \mathrm{GL}_n(\mathbb{C}) \mid g\mathrm{Tr}(g) = 1\},$$
where $\mathrm{Tr}(g)$ is the transpose of the matrix g.

   (a) Show that $\mathrm{O}_n(\mathbb{C})$ is a subgroup of $\mathrm{GL}_n(\mathbb{C})$.

   (b) For $J \in \mathrm{GL}_n(\mathbb{C})$, consider the set
$$\{g \in \mathrm{GL}_n(\mathbb{C}) \mid gJ\mathrm{Tr}(g) = J\}.$$
   Under what conditions on J is this set a subgroup of $\mathrm{GL}_n(\mathbb{C})$ (could be always)?

2. Let S_n be the group of permutations of n objects.

   (a) Recall that the order of an element $w \in S_n$ is the smallest k such that $w^k = 1$. Give an example of an element in S_n with the largest possible order.

   (b) Find a group G such that S_n is a proper subgroup of G.

   (c) Find a group G such that S_n is isomorphic to G/H for some normal subgroup $H \subseteq G$.