Math 3170: Homework 6

1. (a) For which sequence

$$do_n = \#\{\text{distinct partitions of } n \text{ with odd part sizes}\}$$
$$sc_n = \#\{\text{self conjugate partitions of } n\}$$

   is the ordinary generating function easier to directly compute?

   (b) Use Homework 5 to find a generating function for both.

2. For $k \in \mathbb{Z}_{\geq 1}$ compute the coefficients a_n in

$$e^{kx} = \sum_{n \geq 0} a_n \frac{x^n}{n!}$$

   in two ways to show that

$$k^n = \sum_{\substack{m_1 + m_2 + \cdots + m_k = n \\ m_1, m_2, \ldots, m_k \in \mathbb{Z}_{\geq 0}}} \binom{n}{m_1, m_2, \ldots, m_k}.$$

3. Give the first 3 terms of the exponential generating function

$$e^{\frac{e^{tx}-1}{t}}.$$

   (The coefficients in your answer should be polynomials in t). These are known as t-Bell numbers.

4. How many 2-digit positive integers are relatively prime to both 2 and 3?

5. For $m \in \mathbb{Z}_{\geq 1}$, let
$$\phi(m) = \#\{j \in \{1, 2, \ldots, m\} \mid \gcd(m, j) = 1\}.$$

   Let p, q, r be prime numbers. What is $\phi(pqr)$?