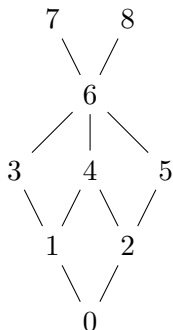# Math 3170: Homework 12

1. Suppose you have a simple graph G with 6 vertices.

   (a) How many edges do you need to guarantee G is connected?
   (b) How many edges do you need to guarantee that every vertex has degree at least 2?

2. Find all posets on $\{1, 2, 3, 4\}$.

3. Consider the poset with Hasse diagram

   (a) Compute $\mu(0, b)$ for all $b \in \{0, 1, 2, \ldots, 8\}$.
   (b) Can you add edges to the Hasse diagram such that $\mu(0, 8) = 2$?

4. Let $\mathcal{P}$ be a poset on a set A. A *chain* in $\mathcal{P}$ of *length* ℓ is a sequence of distinct elements $a_1, \ldots, a_{\ell+1} \in A$ such that $(a_1, a_2), (a_2, a_3), \ldots, (a_\ell, a_{\ell+1}) \in \mathcal{P}$. Let

$$\text{ch}_j = \#\{\text{chains of length } j\}.$$

   Show that for $a, b \in A$ with $a \neq b$,

$$\mu(a, b) = -\text{ch}_1 + \text{ch}_2 - \text{ch}_3 + \cdots.$$