# Math 3170: Homework 9

Due: November 3, 2010

1. How many spanning trees are there of the complete graph $K_n$ that have no vertex with degree greater than 2.

2. The *distance* $d(u, v)$ between two vertices $u$ and $v$ in a connected graph is the smallest number of edges needed to construct a path between $u$ and $v$. The *center* of a connected graph $G$ is the set
$$\{v \in V_G \mid \sum_{u \in V_G} d(u, v) \text{ is minimal}\}.$$
Prove that if $T$ is a tree, then the center of $T$ is either a vertex or a pair of adjacent vertices.

3. How many non-isomorphic trees are there on 7 vertices?

4. Let $G$ be a connected simple graph, and let $S$ and $T$ be spanning trees of $G$.

   (a) Show that if $e \in E_S$, then there exists $f \in E_T$ such that the tree $S'$ obtained by deleting $e$ and adding $f$ is a spanning tree of $G$.

   (b) Show that there is a sequence of spanning trees
   $$S = T_0, T_1, \ldots, T_\ell = T$$
   such that $T_i$ is obtained from $T_{i-1}$ be removing an edge and adding another.

5. Let $G_n$ be obtained from $K_n$ by removing an edge. Find and prove a formula for the number of spanning trees of $G_n$.

   Hint: Count the number of spanning trees of $K_n$ that use the deleted edge.