Math 3170: Homework 7

Due: October 20, 2010

1. Give a self-contained definition of a simple, directed graph.
2. Prove that in a simple graph, if there is a trail between two vertices, then there is also a path between these two vertices.
3. Show that in any simple graph with at least two vertices there are two vertices with the same degree.

   Hint: Focus on a vertex with largest degree, and use the pigeon-hole principle.
4. Prove that the number of people who have shaken hand an odd number of times is even.
5. Is there a non-connected simple graph on 7 vertices with every vertex at least 3?
6. Email me which topic you would like to study for your project.