Math 3140: Homework 9

A. Find an example of a homomorphism that is neither injective nor surjective.

B. 15.2 Find all normal subgroups of D_n.

15.7 Let $K \triangleleft G \times H$ be such that

$$K \cap (\{1_G\} \times H) = \{(1_G, 1_H)\} = K \cap (G \times \{1_H\}).$$

Show that K must be abelian.

15.12 Find a proper normal subgroup of A_4. Show that any non-trivial normal subgroup H of A_5 must contain a 3-cycle, and use 14.5 to conclude that $H = A_5$, thereby proving A_5 is simple.

15.13. Suppose H is a cyclic normal subgroup of G. Show that any subgroup of H is also normal in G.

(2) A group G is *meta-abelian* if there exists and abelian normal subgroup $A \triangleleft G$ such that G/A is also abelian. Show that G is meta-abelian if and only if $[[G, G], [G, G]] = 1$ (the commutator subgroup of the commutator subgroup).

(3) Find the commutator subgroup of $W_{2,n}$.

C. 16.4. Show that if $A \triangleleft G$ and $B \triangleleft H$, then $(A \times B) \triangleleft (G \times H)$ and

$$(G \times H)/(A \times B) \cong (G/A) \times (G/B).$$