Math 3140: Homework 10

A.

15.13. Suppose $N \triangleleft G$ and N is cyclic. Show that any subgroup of N is normal in G.

15.14.
  i. Show that every element in $\mathbb{Q}/\mathbb{Z}$ has finite order.
  ii. Show that the only element in $\mathbb{R}/\mathbb{Q}$ that has finite order is the identity.

B.

16.10.
  i. Find a surjective isomorphism $\varphi : B_n \to S_n$ and conclude that S_n is isomorphic to a quotient of B_n.
  ii. Can you give a description of the kernel of φ in terms of braid diagrams?

16.11. (Fourth Isomorphism Theorem) Let $\varphi : G \to H$ be a surjective homomorphism with kernel K.

  (a) For every subgroup $B \subseteq H$, show that the set

$$\varphi^{-1}(B) = \{g \in G \mid \varphi(g) \in B\}$$

  is a subgroup of G that contains K.

  (b) Show that there is a bijection between

$$\left\{ \begin{array}{c} \text{Subgroups } A \subseteq G \\ \text{such that } K \subseteq A \end{array} \right\} \longleftrightarrow \left\{ \text{ Subgroups } B \subseteq H \right\}.$$

16.12 An *maximal* normal subgroup N of G is a normal subgroup such that if $H \supseteq N$ is a normal subgroup of G, then $H = N$ or $H = G$. Show that N is a maximal normal subgroup of G if and only if G/N is simple.