

# Project 1

**Due: October 10, 2014**

**Recommended length: 2-3 pages.**

**Format: Typed. You will email me a .pdf or .ps file by 12:01pm on the day that the paper is due (no paper copy is necessary)**

The goal of this assignment is to examine the behavior of in and out shuffles. Suppose that you have a deck of  $2n$  cards. Let  $\iota \in S_{2n}$  denote the permutation that gives an in-shuffle, and let  $\omega \in S_{2n}$  denote the permutation that gives the out-shuffle (see Homework 3). We want to begin understanding the subgroup generated by  $\iota$  and  $\omega$ .

The goals of this paper are as follows.

- (1) Identify a characteristic that all permutations in  $\langle \iota, \omega \rangle$  share; make this as precise as possible and prove it. (You are encouraged to check with me whether you've found the "correct" condition).
- (2) Show that all permutations that have the characteristic found in (1) form a *proper* subgroup of  $S_{2n}$ .

Note that you will **not** be showing that every element in  $S_{2n}$  that has the characteristic in (1) is in the subgroup  $\langle \iota, \omega \rangle$  (since it isn't true in general).

In doing this you should

- (a) Briefly introduce the topic.
- (b) Give the necessary definitions and results you will need for your main theorems. You do not need to prove the results that are in the book, but both the results and the definitions should be stated in your own words in a way that focuses them on the theorem. You may assume that the reader has read up through Chapter 8 of the textbook.
- (c) State and prove the main theorem(s).

Note that this is a writing assignment, so the main focus should be on clearly communicating the ideas in the proof. I recommend looking at your favorite mathematics texts and trying to emulate their style. I also suggest you have another member of the class read through a draft before handing it in.

**Extra Credit:** If you type your paper in L<sup>A</sup>T<sub>E</sub>X, then you get 5 points extra credit.