Math 3140: Homework 11

Due: Wednesday, December 10

A. 18.1 Use orbit counting methods to find the number of distinct ways to paint the edges of a cube with two colors.

18.7. How many different ways are there of coloring the vertices and edges of a regular hexagon using red, black or yellow for the edges and black or white for the vertices?

B. 20.1 (a) Show that if $|G| = 126$, then G has a nontrivial proper normal subgroup.
   (b) Show that if $|G| = 1000$, then G is not simple.
   (c) Suppose $|G| = p^k m$ where p is prime and p does not divide m. Prove that if $p > m$, then G is not simple.

(1) Prove that if G is abelian and simple, then $G \cong \mathbb{Z}_p$ for some prime number p.

20.3. (a) Prove that if all the Sylow subgroups are normal, then G is isomorphic to the direct product of of its Sylow subgroups.
   (b) If you know that G is abelian, and $|G| = 154000$, then what do you know about G?

20.7. Classify the groups of order $p^2 q$ if p is not congruent to ± 1 modulo q (and $p \neq q$ are prime).