Math 3140: Homework 3

Due: Wednesday, September 19

A. 6.3. Show that the elements $w \in S_9$ such that $\{w(2), w(5).w(7)\} = \{2, 5, 7\}$ form a subgroup of S_9. What is the order of this subgroup?

6.7+. (a) Describe/characterize the elements of order 2 of S_n.

(b) Show that if $n \geq 4$, then every permutation can be written as a product of two permutations of order 2. Hint: Answer the question first for cyclic permutations.

(c) What goes wrong if $n < 4$?

B. 7.5. Let G be a group. Show that the function

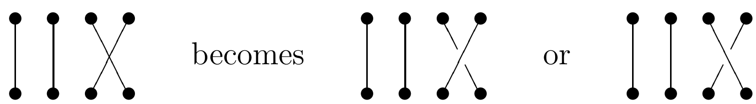
$$\begin{array}{rcl} \varphi : G & \longrightarrow & G \\ x & \mapsto & x^{-1} \end{array}$$

is an isomorphism if and only if G is abelian.

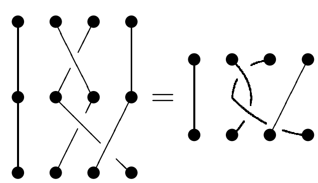
7.9. Suppose G is cyclic with generator $x \in G$. Show that if $\varphi : G \to H$ is an isomorphism, then φ is completely determined by $\varphi(x)$. Show that $H = \langle \varphi(x) \rangle$.

C. (1) Show that there are exactly two groups with four elements (up to isomorphism).

(2) The *braid group* B_n is a group generated by the diagrams of S_n but we keep track of where strings cross. For example,



and we keep track of these crossings when multiplying,



What is the inverse of an element in B_n? Show that B_n has infinite order. What are the elements of finite order in B_n?