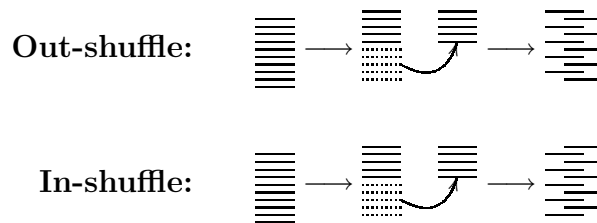


Math 3140: Homework 3

Due: Wednesday, September 19

- A. Find examples of the following (justify your answer), or explain why they don't exist:
- (1) A nonabelian cyclic group.
 - (2) A group generated by two elements.
 - (3) A group generated by three elements, but not by two element.
- B. Show that $(\mathbb{Q}, +)$ is not cyclic. In fact, show that $(\mathbb{Q}, +)$ does not even have a finite number of generators. Hint: What does the subgroup $\langle p/q \rangle$ look like for $p, q \in \mathbb{Z}_{\neq 0}$?
- C. (1) Develop an argument (as mathematically precise as possible) for why S_3 and D_3 are really the same group.
- (2) Suppose you have a group with order 6. Show that you can't have elements of order 4 or 5 in the group. Show that you can have elements with order 1, 2, 3 and 6 (Find specific examples).
- D. A *perfect shuffle* is a shuffle where you split the deck into two equal stacks (the top stack and the bottom stack) and push the stacks together in such a way that the cards from the two stacks alternate. There are two ways of doing this: either the bottom card came from the top stack (an *in-shuffle*), or the bottom card came from the bottom stack (an *out-shuffle*). For example, if you have 10 cards,



By a judicious use of in and out-shuffles one can create all kinds of card tricks (providing one is capable of a perfect shuffle), since such shuffling is in no way random. Suppose you have a deck of 52 cards.

- (1) Prove that if you apply the in-shuffle enough times, you will get back with what you started. Hint: prove that you get back where you started without actually doing it (it's a lot of shuffles).
- (2) How many out-shuffles do you need to get back where you started (Justify your answer)?

Remark: It turns out with normal shuffling (which is far from perfect) it takes approximately 7 shuffles to get close to random (which is proved in part using methods of this class).