Math 3140: Homework 11

Due: Wednesday, December 5

A. 17.1. Let $G = \langle (1,2,3)(4,5), (7,8) \rangle \subseteq S_8$. Then G acts on the set $X = \{1,2,3,4,5,6,7,8\}$. Calculate the orbits of the G-action in X, and the stabilizer of each element.

17.7-8. **The diagonal action.** Suppose G acts on X and Y. Let

$$\begin{array}{ccc} G \times (X \times Y) & \longrightarrow & X \times Y \\ (g, (x, y)) & \mapsto & (g(x), g(y)) \end{array} \qquad (*)$$

i. Show that $(*)$ gives an action of G on $X \times Y$.

ii. Find the stabilizer of $(x, y) \in X \times Y$.

iii. Give an example to show that this action is not necessarily transitive, even if G acts transitively on both X and Y.

iv. Find the orbits and stabilizers of $G = \langle (1,2,3,4), (2,4) \rangle \subseteq S_4$ acting on $X \times X$ diagonally, where $X = \{1,2,3,4\}$.

17.10. Recall that if $x \in G$, then the stabilizer of G in X is

$$C_G(x) = \{g \in G \mid gxg^{-1} = x\}.$$

(This is the stabilizer of an element under the conjugation action of G on $X = G$). Show that if some conjugacy class of G has exactly two elements, then G is not simple.

B. 18.1 Use orbit counting methods to find the number of distinct ways to paint the edges of a cube with two colors.

18.7. How many different ways are there of coloring the vertices and edges of a regular hexagon using red, black or yellow for the edges and black or white for the vertices?