

Math 2001: PHW4

1. From the book do:

- 2.10: 2, 4, 6, 8, 10
- 5: 2, 4, 14

2. Identify whether each of the following statements is true or false. If it is true, prove it. If it is false, then provide a counterexample.

(a) Let A , B , and C be sets. Then

$$(A \cap B) \cup C = A \cap (B \cup C).$$

(b) If $a, b \in \mathbb{Z}_{\geq 1}$ and both \sqrt{a} and \sqrt{b} are irrational, then \sqrt{ab} is irrational.

3. A point (m, n) in \mathbb{R}^2 is a *lattice point* if both $m, n \in \mathbb{Z}$. Prove that the number of lattice points inside any circle centered at the origin is a number of the form $4k + 1$ for some integer k (note that you don't have to say what k is).

Hint: Split the set of lattice points into subsets, depending on the quadrants.