Math 2001: PHW7

1. Consider the following

   Claim. *The number $n(n+1)$ is an odd number for every n.*

   Proof. Assume the statement is true for n. We prove the statement for $n+1$ by induction. Note that
   $$(n+1)((n+1)+1) = n(n+1) + 2(n+1).$$
   By induction $n(n+1)$ is odd. Thus, $(n+1)((n+1)+1)$ is the sum of an odd number $n(n+1)$ and an even number $2(n+1)$. The sum of an odd number and an even number is odd. Thus, we have proved the claim by induction. $\square$

   I checked the claim and it doesn't seem to work for $n = 15$, since $15 \cdot 16 = 240$, which is even. What is wrong with the proof?

2. Six poker players each start with \$5. After an evening of play where all bets are multiples of \$.10, how many different ways could the funds be split up?

3. Our class has 24 registered students. Each student will get an A, B, C, D, or F (we assume there are no Ws or Is).

   (a) How many ways are there to assign the grades to the class (and no, you may not assume you get an A)?

   (b) Suppose I need to report my grade distribution (how many As, how many Bs, etc) to my department; how many possible grade distributions are there for this class?

   (c) Suppose that for every grade, there is at least one student who received that grade. How many grade distributions are there now?

4. For each of the following sets, count the number of 4 letter words that one can make using letters from the set.

   (a) The set $\{A, B, C, D, E, F, G, H, I\}$ ($ABCD$ is valid, but $AABC$ is not).

   (b) The multi-set $\{T, E, L, E, P, H, O, N, E\}$ ($EELE$ is valid, but $EEEE$ is not).

5. In poker, the more likely a hand is to appear, the less valuable it is. For the following 5 card hands, determine their probability of appearing, and then order the hands by value:

   (a) 5-card flush

   (b) four of a kind

   (c) full house

   (d) 5-card straight

6. A 5 digit number contains no zeroes. What is the probability that it has exactly three distinct digits?