Math 2001: Homework 12

Due: December 3, 2008

Give complete justifications for all your answers.

Problem 1

1. Consider the sequence given by

$$a_n = 3a_{n-1} - 2a_{n-2}, \qquad \text{with } a_1 = 3, a_2 = 7.$$

   Prove that $a_n = 2^{n+1} - 1$. Hint: Use induction.

2. Let $A_1, A_2, \ldots, A_n \subseteq X$ be subsets of a big set X. Prove that

$$(A_1 \cap A_2 \cap \cdots \cap A_n)^c = A_1^c \cup A_2^c \cup \cdots \cup A_n^c$$

   in two ways: use both a proof by induction and a direct proof.

   Hint: For the direct proof, show that $LHS \subseteq RHS$ and $LHS \supseteq RHS$.