Math 2001: Homework 10

Due: November 12, 2008

Give complete justifications for all your answers.

Problem 1

1. Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is injective but not bijective. (Be sure to prove both assertions).

2. Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is surjective but not bijective. (Be sure to prove both assertions).

3. For $n, k \in \mathbb{Z}_{\geq 0}$, let X_k be the set of subsets of $\{1, 2, \ldots, n\}$ with k elements. Define

$$f : X_k \to X_{n-k}$$

by $f(A) = A^c$ for $A \in X_k$ (you should first make sure this even makes sense). Find an inverse map for f (thereby showing that this map is a bijection). What does this say about Pascal's triangle?

Problem 2

From 3.2 problems...

1. The genetic code can be viewed as a sequence of four letters T, A, G, and C. How many six-letter sequences are there?

2. How many of these six letter sequences are palindromic (that is, they are the same when read in the reverse order)?

Problem 3

A normal deck of cards has 52 different cards, so in general there are

$$\binom{52}{5} = 2,598,960$$

different possible poker hands (the number of subsets of size 5 of a set of size 52). This problem asks for the probability that one obtains various kinds of poker hands. In general, the probability that $BLAH$ happens is given by

$$\frac{\text{the number of ways that } BLAH \text{ can happen}}{\text{the total number of things that could possibly happen}}.$$

For example, the probability that you get four aces is

$$\frac{48}{2598960} = \frac{1}{54145} \approx 00.002\%$$

since there are exactly 48 ways one can be dealt a hand with four aces (one gets 4 aces plus some other card). For the following problems be sure to justify your answer, since I imagine you can look up the answers from a variety of sources.

1. What is the probability of getting four of a kind (this should be more likely than getting 4 aces)?

2. What is the probability of getting a flush (all five cards in the same suit)?

3. What is the probability of getting a straight (5 cards in a sequence, but not necessarily in the same suit. The ace can be high or low, but not in the middle of a sequence)?