

Worksheet 8: Conjugacy classes of S_n

The *cycle type* of a permutation $w \in S_n$ is the sequence of cycle lengths of its cycle decomposition, listed in non-increasing order.

1. In S_9 , find the cycle type of $w = (1, 5, 3, 4)(2, 6, 9)$, $v = (1, 3)(5, 9, 8, 2)(6, 7)$ and v^{-1} (don't forget the 1-cycles).

2. With v, w as in 1, find the cycle type of vwv^{-1} .

3. With v as in 1, find

$$v(1, 8, 4, 5, 9, 3)v^{-1}.$$

4. For $v \in S_n$ and $(i_1, i_2, \dots, i_\ell)$ an ℓ -cycle in S_n , show that

$$v(i_1, i_2, \dots, i_\ell)v^{-1} = (v(i_1), v(i_2), \dots, v(i_\ell))$$

by showing that both give the same permutation of $\{1, 2, \dots, n\}$.

5. Explain why 1 implies that v and w can only be conjugate if they have the same cycle type.
6. Show that if v and w have the same cycle type, then they are conjugate.
7. How many permutations are in the conjugacy class corresponding to the cycle types $(2, 1, \dots, 1)$ and (n) ?
8. Show that $Z(S_n) = \{1\}$.