Worksheet 5: The coin-stack group

Consider a stack of n 2-sided coins. A symmetry of the stack will be a shuffle of the stack, where one can also flip over the coins. Let

$$WB_n = \{\text{symmetries of a stack of } n \text{ coins}\}$$

be the corresponding group of symmetries.

1. What is the order of WB_n?
2. Make the case that WB_n is isomorphic to a subgroup of S_{2n} (not Cayley's Theorem).
3. Show that S_n is isomorphic to a subgroup of WB_n.
4. Find another subgroup $H \subseteq WB_n$ such that
   - $S_n \cap H = \{1\}$
   - $WB_n = H \cdot S_n$.
5. What is H isomorphic to?
6. Why is $WB_n \not\cong H \times S_n$?
7. Find an element $h \in H$ such that $WB_n = \langle h, (1,2), (2,3), \ldots, (n-1, n) \rangle$.

Individual write-up (due October 11, 2019): Write up a narrative that answers 3–6 (should not be just a list of answers). State results where appropriate and give their proofs.