Worksheet 4: Cayley's Theorem

For $g \in G$, let

$$\begin{array}{rccc} L_g: & G & \longrightarrow & G \\ & h & \mapsto & gh. \end{array}$$

1. Show that $L_g \in S_G$.

2. Write down the functions $L_2 \in S_{\mathbb{Z}_3}$ and $L_s \in S_{D_4}$.

$$\begin{array}{rccc} L_2: & \mathbb{Z}_3 & \longrightarrow & \mathbb{Z}_3 \\ & 0 & \mapsto & \\ & 1 & \mapsto & \\ & 2 & \mapsto & \end{array}
\qquad
\begin{array}{rccc} L_s: & D_4 & \longrightarrow & D_4 \\ & 1 & \mapsto & \\ & r & \mapsto & \\ & r^2 & \mapsto & \\ & r^3 & \mapsto & \\ & s & \mapsto & \\ & rs & \mapsto & \\ & r^2s & \mapsto & \\ & r^3s & \mapsto & \end{array}$$

Consider the function

$$\begin{array}{rccc} L: & G & \longrightarrow & S_G \\ & g & \mapsto & L_g. \end{array} \tag{$*$}$$

and let

$$L(G) = \{L_g \mid g \in G\}.$$

3. Show that $L(G)$ is a subgroup of G.

4. Show that $L : G \to L(G)$ is an isomorphism.

You have shown that G is isomorphic to a subgroup of S_G. Since $S_G \cong S_{|G|}$ we also have that G is isomorphic to a subgroup of $S_{|G|}$.

5. Find the copy cycle decompositions of the permutations of D_4 sitting inside of S_8 using $(*)$ and an isomorphism from Monday.