Worksheet 2: Dihedral groups

Given a set of points $C \subseteq \mathbb{R}^2$, the *convex polygon* induced by C is the smallest set of points $\mathrm{pol}(C) \subseteq \mathbb{R}^2$ such that $C \subseteq \mathrm{pol}(C)$, and for all $a, b \in \mathrm{pol}(C)$,

$$a + (1-t)b \in \mathrm{pol}(C), \quad \text{for all } 0 \leqslant t \leqslant 1. \tag{$*$}$$

1. Draw a picture of $\mathrm{pol}(\{(0,0),(1,0),(1,1)\})$.
2. What's the deal with $(*)$?

The regular n-gon pol_n is the convex polygon induced by the set

$$V_n = \{(1; 2\pi j/n) \mid 0 \leqslant j \leqslant n-1\},$$

where $(r;\theta)$ is written in polar coordinates.

3. Draw pictures of pol_4, pol_5 and pol_6.
4. How many symmetries does pol_n have? Why?

The group of symmetries of pol_n is called the *dihedral group* D_n.