

Worksheet 13: Sylow theory

Goal 1. Groups of order pq where p and q are prime.

1. Show that there are no simple groups of order 15 and 21.
2. Generalize to show that there are no simple groups of order pq for $p < q$ prime.
3. Why do we already know this for groups of order p^2 ?

Goal 2. Groups G of order 30.

4. If there is no normal Sylow 5-subgroup, how many elements of order 5 does G have?
5. If there is no normal Sylow 3-subgroup, how many elements of order 3 does G have?
6. Discover the contradiction!

Goal 3. Groups G of order p^2q with p, q prime.

7. Show that if G has no normal Sylow q -subgroup, then it must have a normal p -subgroup (will involve cases).

Group write-up. Show that if G is a non-abelian simple group of order ≤ 30 , then $|G| = 24$ (in fact, there is no simple group of order 24, but one needs slightly fancier techniques to see this).