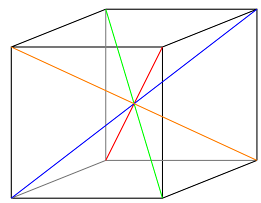
Worksheet 12: Orbit counting

On Monday we worked out the conjugacy class sizes of S_4,

conjugacy class $\mathcal{C}$	$(1,1,1,1)$	$(2,1,1)$	$(2,2)$	$(3,1)$	(4)
$\lvert\mathcal{C}\rvert$	1	$\binom{4}{2}$	$\binom{3}{1}$	$\binom{4}{1}2!$	$3!$

Goal 1. Show that S_4 is the group of rotational symmetries of a cube.

1. Count the number of rotational symmetries of the cube.
2. Note that each rotational symmetry permutes the diagonals of the cube



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Number the diagonals and show how to find the permutations $(1,2)$, $(2,3)$ and $(3,4)$.

3. For each conjugacy class of S_4 determine how a representative (you choose which) rotates the cube.

Goal 2. Suppose you color the faces of the cube with red, blue, green or yellow. How many different ways are there of doing this?

4. Ignoring rotational symmetry how many different ways are there to color the faces of the cube (this is the size of the set A of all colorings)?
5. For each conjugacy class representative w from 3, determine $\lvert\mathrm{Fix}_A(w)\rvert$.

conjugacy class $\mathcal{C}$	$(1,1,1,1)$	$(2,1,1)$	$(2,2)$	$(3,1)$	(4)
$\lvert\mathrm{Fix}_A(w)\rvert$					

6. How many different ways are their to color the cube where we consider two colorings to be the same if you can rotate one into another?