

## Math 3140: Homework 5

A. 7.2 Verify that the set of integers  $\{1, 3, 7, 9, 11, 13, 17, 19\}$  form a group under multiplication modulo 20. Explain why this group is not isomorphic to  $\mathbb{Z}_8$ .

7.5 Show that the function

$$\begin{aligned} \varphi : G &\longrightarrow G \\ g &\longmapsto g^{-1} \end{aligned}$$

is an isomorphism if and only if  $G$  is abelian.

7.9 Suppose  $G$  is cyclic with generator  $x \in G$ . Show that if  $\varphi : G \rightarrow H$  is an isomorphism, then  $\varphi$  is completely determined by  $\varphi(x)$ . Show that  $H = \langle \varphi(x) \rangle$ .

B. 8.7 Using Cayley's Theorem, explicitly find an isomorphic copy of  $D_3$  inside  $S_6$ .

8.10 For each  $w \in S_n$ , let  $\tilde{w} \in S_{2n}$  be the permutation given by

$$\tilde{w}(j) = \begin{cases} w(j), & \text{if } 1 \leq j \leq n, \\ w(j-n) + n, & \text{if } n+1 \leq j \leq 2n. \end{cases}$$

(a) Describe the relationship between the cycle decomposition of  $w$  and the cycle decomposition of  $\tilde{w}$ .

(b) Show that the function that sends  $w \mapsto \tilde{w}$  is an isomorphism between  $S_n$  and a subgroup of  $A_{2n}$ .

8.11 Let  $G$  be the full symmetry group of a regular tetrahedron  $T$ , and adopt the notation of Figure 7.2 in the book. Find the symmetry  $q$  of  $T$  which induces the transposition  $(1, 2)$  of the vertices, and show that  $qr$  induces the 4-cycle  $(1, 2, 3, 4)$ . Check that  $qr$  is neither a rotation nor a reflection, but is the product of three reflections. Count the symmetries of  $T$  and prove that  $G$  is isomorphic to  $S_4$ .