

Math 3140: Homework 11

17.1. Let $G = \langle (1, 2, 3)(4, 5), (7, 8) \rangle \subseteq S_8$. Then G acts on the set $X = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Calculate the orbits of the G -action in X , and the stabilizers of 1, 2 and 7.

17.4 Let G act on a set X , and let O be an orbit in G . Show that $\text{Stab}_G(x) = \text{Stab}_G(y)$ for all $x, y \in O$ if and only if $\text{Stab}_G(x) \triangleleft G$.

17.7-8. **The diagonal action.** Suppose G acts on X and Y . Let

$$\begin{aligned} G \times (X \times Y) &\longrightarrow X \times Y \\ (g, (x, y)) &\mapsto (g(x), g(y)) \end{aligned} \quad (*)$$

- (a) Show that $(*)$ gives an action of G on $X \times Y$.
- (b) For example, find the orbits of $G = \langle (1, 2, 3, 4), (2, 4) \rangle \subseteq S_4$ acting on $X \times X$ diagonally, where $X = \{1, 2, 3, 4\}$. Is this action transitive?
- (c) In general, find the stabilizer of $(x, y) \in X \times Y$ (in terms of the stabilizers of x and y).

17.10. The *centralizer* of $g \in G$ is the subgroup

$$C_G(g) = \{h \in G \mid hgh^{-1} = g\}.$$

- (a) Identify a set X with an action of G on that set such that $C_G(g) = \text{Stab}_G(g)$ for that action.
- (b) Show that if some conjugacy class of G has exactly two elements, then G is not simple.