

Review for Exam 1

Recall that your exam is on Wednesday, February 20 (Section 001: Thursday, February 21) in class. Calculators WILL be allowed on the exam. No notes or crib sheets or texts or other references will be allowed. If you understand and can do all assigned homework problems, all in-class worksheet problems, and all problems on this handout, you should do fine.

IMPORTANT NOTE: on the exam, you will be asked to SHOW YOUR WORK. Get into the habit of doing so here, if you have not already.

1. Suppose an elevator in the Empire State Building is programmed to move only one floor at a time. Also suppose that, each time it moves from a given floor, it has probability $p = .7$ of moving up to the next higher floor, and probability $q = .3$ of moving down to the next lower floor.

(a) If the elevator begins at the 50th floor, find the probability that, after 10 moves, it ends up on the 46th floor.

(b) If the elevator begins at the 50th floor, find the probability that, after 10 moves, it ends up on the 56th floor.

2. (a) In how many ways can the letters in the word WIDGET be rearranged?

(b) How many 4-letter sequences can be made from the letters in WIDGET?

3. Suppose this class has 35 students.

(a) How many different 5-person basketball teams can be made from the members of the class?

(b) In how many ways can a center, left forward, right forward, right guard, and left guard be chosen?

4. Suppose Shaq's free-throw percentage is 55%. If he takes 10 free throws in a game,

(a) What's the probability that he makes exactly 6 of them?

(b) What's the probability that he makes 9 of them?

(c) What's the probability that he makes at least 9 of them?

5. How many different license plates are possible if each plate must consist of three digits (0–9), with no repetitions, followed by three letters (A–Z), with no repetitions?

6. What's the probability of winning a lottery in which 5 numbers are randomly selected out of 48?

7. Show, without using your calculator, that:

(a) ${}_6C_4 = 15$;

(b) ${}_{12}C_5 = 11 \cdot 9 \cdot 8$;

(c) ${}_{54}C_4 = 27 \cdot 53 \cdot 13 \cdot 17$;

(d) ${}_{1001}C_5 = 1001 \cdot 25 \cdot 333 \cdot 998 \cdot 997$.

21. A card is selected at random from a standard deck of 52 cards. Find the probability that:

(a) the card is a face card;

- (b) the card is a diamond;
 - (c) the card is either a face card or a diamond (or both);
 - (d) the card is either a face card or a diamond but *not* both;
 - (e) the card is neither a face card nor a diamond.
8. Suppose A and B are independent, $P(A) = 0.6$, and $P(B) = 0.3$. Find $P(A \cup B)$.
9. An *unfair* coin, for which $P(\text{heads}) = \frac{2}{3}$, is flipped 3 times.
- (a) Draw a tree diagram for the three flips.
 - (b) Find $P(\text{no heads})$, $P(\text{exactly one head})$, $P(\text{exactly two heads})$, $P(\text{three heads})$.
 - (c) Find the expected value E of the number of heads.
 - (d) Find odds in favor of, and against, getting fewer than two heads.
10. A box contains 4 black and 3 white marbles. One marble is drawn at random from the box and *not* replaced; then another marble is drawn at random.
- (a) Draw a tree diagram for the two draws.
 - (b) Find $P(\text{the two marbles have the same color})$, and $P(\text{they have different colors})$.
 - (c) Find the expected value E of the number of black marbles.
 - (d) Find the odds in favor of, and against, getting all black marbles.
11. Repeat problem 10 but this time sampling *with* replacement: that is, the first marble is put back in the box before the second one is drawn.
12. You pay me a fee to enter a game where I roll a fair die, and I pay you \$2 if a 1 or a 2 comes up, \$1 if a 3, 4, or 5 comes up, and \$7 if a 6 comes up. In order that it be a fair game, how much should the fee be?
13. You and I play a game where I roll a fair die, and if a 1, 2, 3, 4, or 5 comes up, I pay you the dollar amount shown on the die. In order that the game be fair, how much should you pay me if a 6 comes up?
14. The probability that a randomly selected CU student likes Nine Inch Nails is 0.1. The probability that a randomly selected CU student likes Nine Inch Nails AND *NSYNC is 0.01. If a CU student chosen at random is known to like Nine Inch Nails, what's the probability that she/he likes *NSYNC?
15. If four people are chosen at random, what's the probability that at least two were born on the same day of the week?
16. (a) According to Sportfanatik.com, the odds in favor of the Philadelphia Eagles winning the 2009 National Football Conference Championship are 1:7. If this is accurate, what is the probability of them winning this championship?
- (b) According to Sportfanatik.com, the odds in favor of the New York Giants winning the 2009 National Football Conference Championship are 2:7. If this is accurate, what is the probability of them winning this championship?
- (c) Since the odds in favor of the Philadelphia Eagles winning the 2009 National Football Conference Championship are 1:7, and the odds in favor of the New York Giants winning the 2009 National Football Conference Championship are 2:7, the Giants are twice as likely to win. Right?