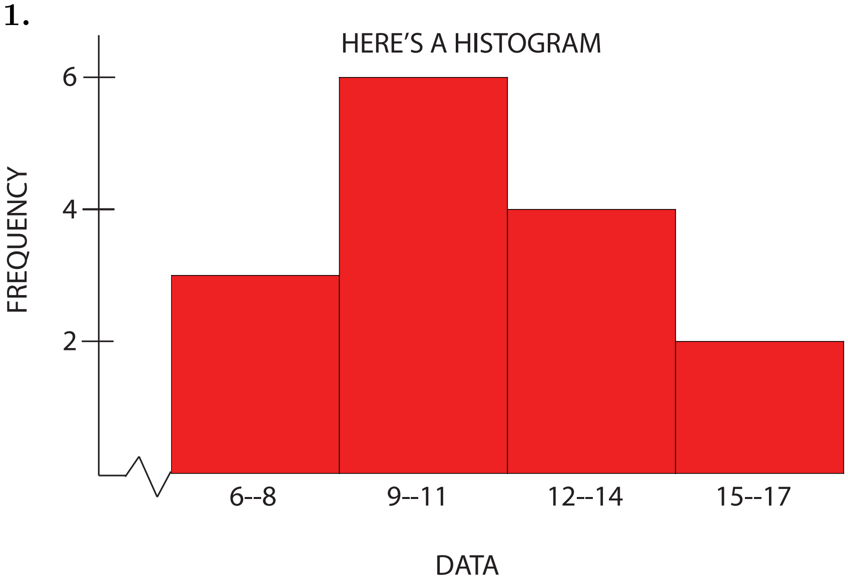
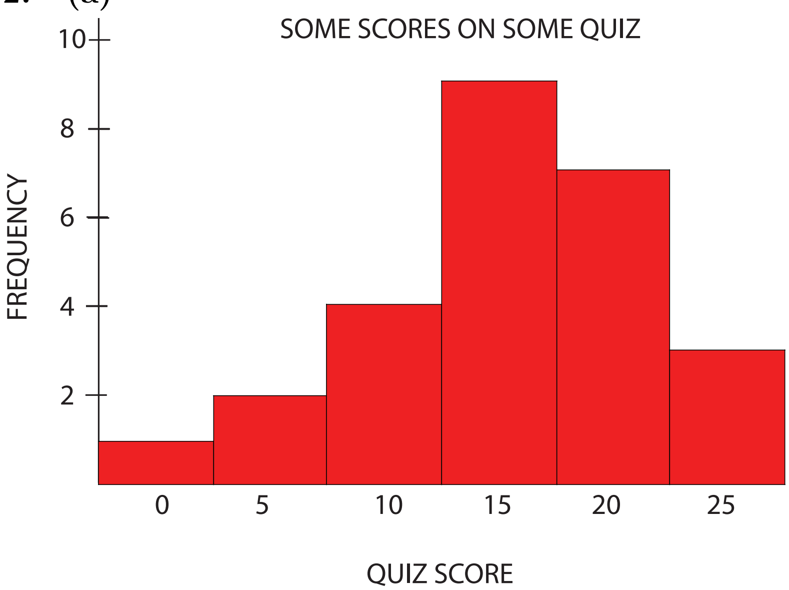
Math 1120: The Spirit and Uses of Mathematics **Spring 2007**

Review for Exam 2: ANSWER KEY

1.



2. **(a)**



(b) $\overline{x} = 15.3846$, $s = 6.1898$ **(c)**76.9%, 96.2%, 100%

3. A; A; A; S; N; S; S.

4. 8.5

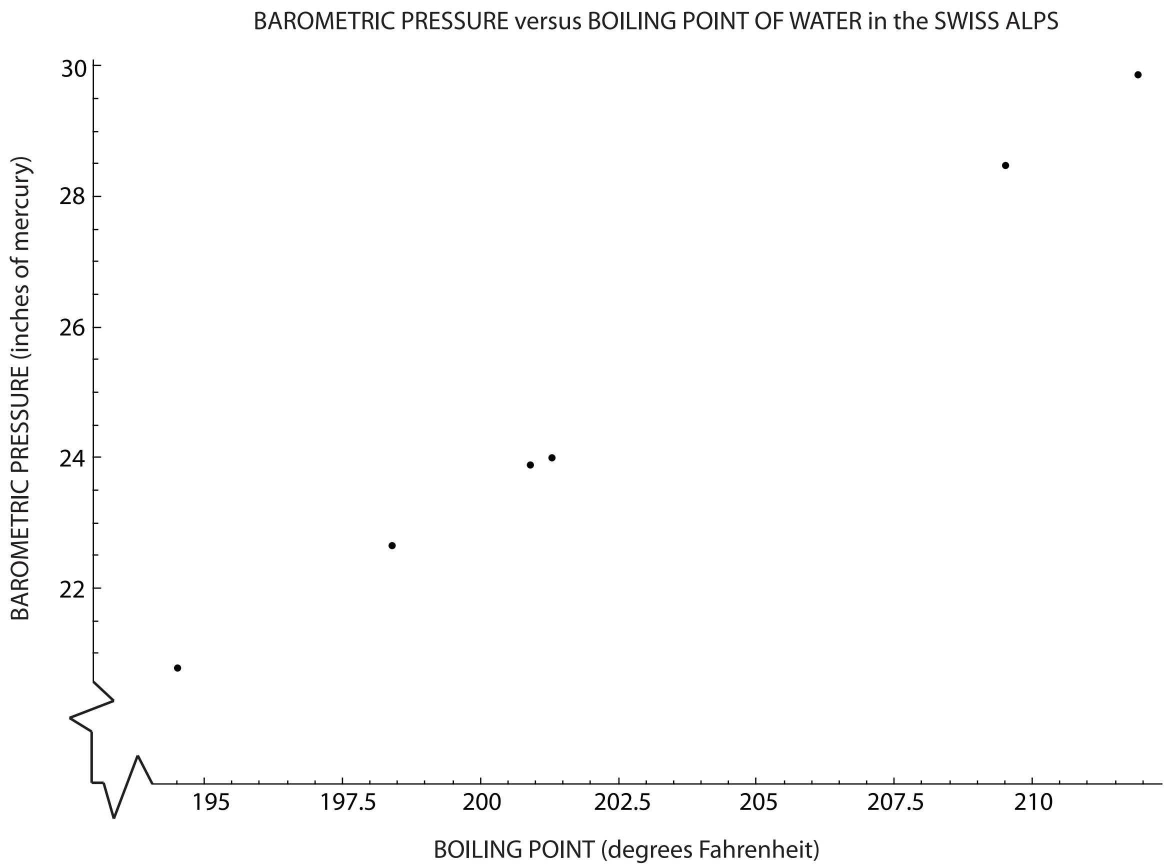
5. **(a)** 0.00900169 **(b)** 0.622599

6. P; P; P; P; I

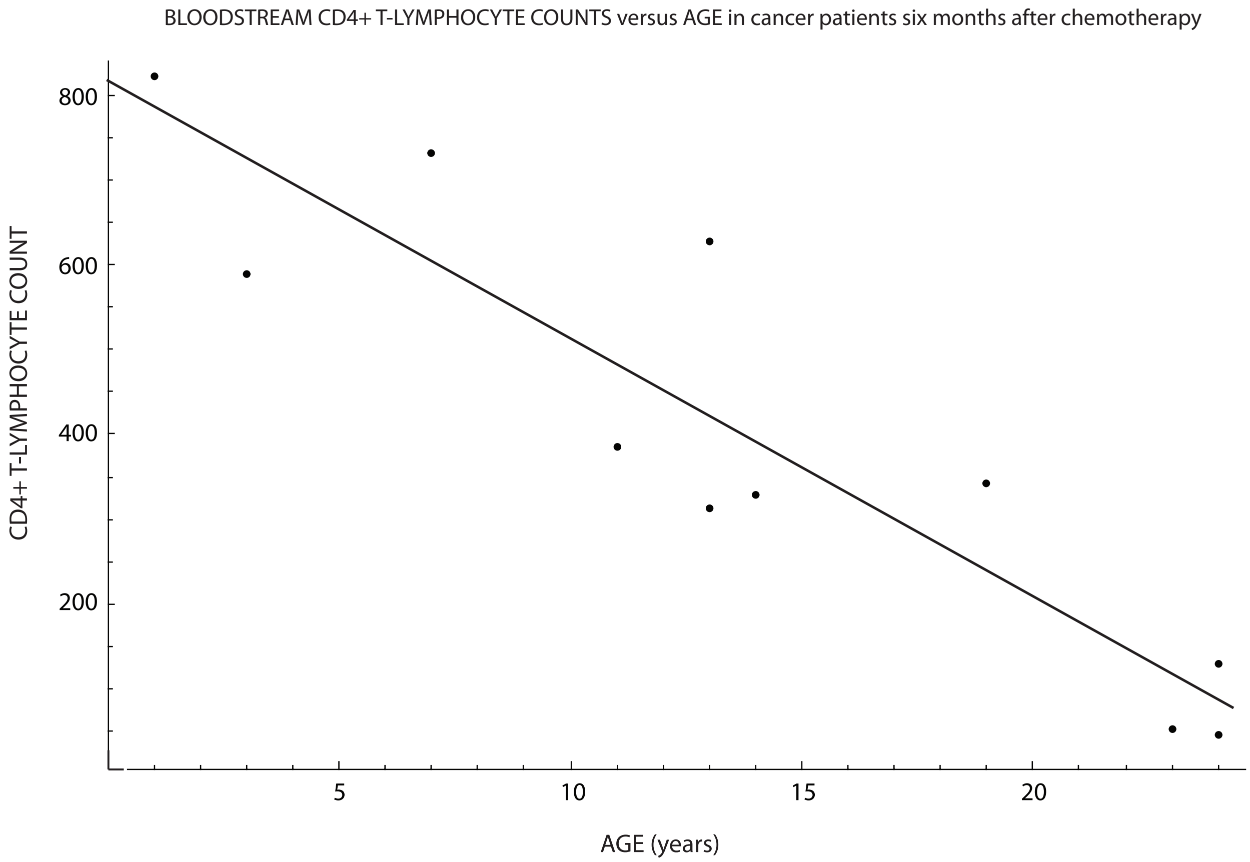
7. **(a)** 5.5 **(b)** 5.5 **(c)** The standard deviation will probably be larger in the first case, for the following reason. All numbers on the die are equally likely, so numbers FAR from the mean of 5.5 are just as likely as those CLOSE to the mean, so the data is quite spread out. On the other hand, if you flip a fair coin 11 times, then only RARELY will you get zero heads, or 11 heads, or 1 head, or 10 heads: numbers close to the mean of 5.5 are MORE likely than those far away. So the data is less spread out from the mean, so the standard deviation should be smaller.

8. Neither should change much. The mean should be about 5.5 after a large number of rolls; it shouldn't change much as you roll more and more. Nor should the spread of the data, so the standard deviation shouldn't change much either.

9. The graph is showing that boiling point increases as atmospheric pressure increases.



10. Negative: as age increases, CD4+ T-lymphocyte count six months after chemotherapy decreases. 220 or so. About 14.



11. **(a)** $6! = 720$ **(b)** ${}_6P_4 = 6 \cdot 5 \cdot 4 \cdot 3 = 360$

12. **(a)** ${}_{40}C_5 = 658008$ **(b)** ${}_{40}P_5 = 78960960$

13. **(a)** 0.238367 **(b)** 0.0207241 **(c)** 0.0232571

14. $10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24 = 11232000$

15. $1/{}_{48}C_5 = 1/1712304 = 5.84 \cdot 10^{-7} = 0.0000584\%$

16. **(a)** ${}_6C_4 = \dfrac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{6 \cdot 5}{2 \cdot 1} \cdot \dfrac{4 \cdot 3}{4 \cdot 3} = \dfrac{30}{2} = 15$;

(b) ${}_{12}C_5 = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{12}{4 \cdot 3} \cdot 11 \cdot \dfrac{10}{5 \cdot 2} \cdot 9 \cdot 8 = 11 \cdot 9 \cdot 8$;

(c) ${}_{54}C_4 = \dfrac{54 \cdot 53 \cdot 52 \cdot 51}{4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{54}{2} \cdot 53 \cdot \dfrac{52}{4} \cdot \dfrac{51}{3} = 27 \cdot 53 \cdot 13 \cdot 17$;

(d) ${}_{1001}C_5 = \dfrac{1001 \cdot 1000 \cdot 999 \cdot 998 \cdot 997}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1001 \cdot \dfrac{1000}{5 \cdot 4 \cdot 2} \cdot \dfrac{999}{3} \cdot 998 \cdot 997$
$= 1001 \cdot 25 \cdot 333 \cdot 998 \cdot 997$.

17. 79.8679

18. 87

19. The new median is still 77. Why? because two of the four late students scored above 77 and two below; so after these four scores are included, we still have as many scores below the score of 77 as we do below.

20. 1, 1, 2, 4, 8, 8. The second part is impossible, and here's why. In a data set of 6 data points, the median is midway between the third and fourth largest values. But if the median equals a mode, then those third and fourth values must EQUAL each other (because at least two data points must equal the mode). So our data set must look like this: x, y, O, O, u, v, where O stands for "mode." Now since the data is bimodal, we must either have $x = y$ or $u = v$: that is, our data set either looks like x, x, O, O, u, v or x, y, O, O, u, u. But in the first case the mean is clearly larger than x, so the mean cannot equal the other mode. In the second case the mean is clearly less than u, so the mean cannot equal the other mode.