

Solutions to some practice problems
Calc 2, Midterm 2, Spring 2007

1. (a) $y = \frac{\sinh^{-1}(x)+C}{\sqrt{1+x^2}}$
(b) $y = Ke^{x^3/3} - 2$
(c) $y = \frac{K}{e^{\sin(x)}} + 1$
(d) $y = K\sqrt{x^2 + 1}$
2. (a) Linearly independent solutions: $y = e^x, y = 1$ (other answers may be correct)
General solution: $y = C_1e^x + C_2$
(b) Linearly independent solutions: $y = e^{4x}, y = xe^{4x}$
General solution: $y = C_1e^{4x} + C_2xe^{4x}$
(c) Linearly independent solutions: $y = e^{\left(\frac{-3+\sqrt{13}}{2}\right)x}, y = e^{\left(\frac{-3-\sqrt{13}}{2}\right)x}$
General solution: $y = C_1e^{\left(\frac{-3+\sqrt{13}}{2}\right)x} + C_2e^{\left(\frac{-3-\sqrt{13}}{2}\right)x}$
(d) Linearly independent solutions: $y = \sin(10x), y = \cos(10x)$
General solution: $y = C_1 \sin(10x) + C_2 \cos(10x)$
3. (a) $a_n = 2 + (-1)^n$, limit does not exist
(b) $a_n = \frac{2n-1}{(n+1)(n+2)}$, limit = 0
(c) $a_n = \frac{3 \cdot (-2)^{n-1}}{n^2+1}$, limit does not exist
4. (a) $\frac{1}{22}$
(b) $-2.32 - \frac{7}{900} = -\frac{419}{180}$
(c) $\frac{1+\sqrt{5}}{2}$
5. Note: for (c), (d), and (e), the limits are actually pretty hard to find. Don't sweat it if you couldn't find them (you need to use the "Squeezing Theorem".)
(a) Eventually increasing; limit = 1
(b) Increasing; limit = 1
(c) Decreasing; limit = 0
(d) Eventually increasing; limit does not exist (it goes to ∞)
(e) Decreasing; limit = 0
(f) Increasing; limit = 0

6. (a) $\frac{2^n-1}{n}$; limit = ∞
(b) $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} \right)$; limit = $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{3} \right) = \frac{5}{12}$
7. $20 \left(\frac{1}{2} \right)^{2/5} \approx 15.16$ lbs of salt (there are $20 \left(\frac{200}{200+25t} \right)^{2/5}$ lbs of salt at time t .)
8. (a) This one diverges.
(b) $1/48$
(c) $3/5$
(d) $-3/4$
(e) $3/4$