

Some practice problems
Calc 2, Midterm 2, Spring 2007

1. Find the family of solutions for each of the following differential equations. If possible, solve for y .

(a) $(1 + x^2) \frac{dy}{dx} + xy = 1$

(b) $\frac{dy}{dx} - x^2y = 2x^2$

(c) $\frac{dy}{dx} + y \cos(x) = \cos(x)$

(d) $(1 + x^2) \frac{dy}{dx} = xy$

(Try solving this last one two different ways. Do your answers match?)

2. Find two linearly independent solutions to each of the following differential equations, and give a formula for the general solution.

(a) $\frac{d^2y}{dx^2} = \frac{dy}{dx}$

(b) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$

(c) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$

(d) $\frac{d^2y}{dx^2} + 100y = 0$

3. Write a formula for each of the following sequences, and determine if it has a limit. If it has a limit, find it.

(a) 1, 3, 1, 3, 1, 3, ...

(b) $\frac{1}{2 \cdot 3}, \frac{3}{3 \cdot 4}, \frac{5}{4 \cdot 5}, \frac{7}{5 \cdot 6}, \dots$

(c) $\frac{3}{1+1}, -\frac{6}{4+1}, \frac{12}{9+1}, -\frac{24}{16+1}, \dots$

4. Determine the value of each of the following.

(a) .0454545454545...

(b) -2.32777777777...

(c) $\sqrt{2 + \frac{1}{\sqrt{2 + \frac{1}{\sqrt{2 + \frac{1}{\dots}}}}}}$

Hint: $L^3 - 2L - 1 = (L + 1)(L^2 - L - 1)$

5. Determine if each of the following sequences is eventually increasing or eventually decreasing, and find its limit.

(a) $\left\{ \frac{t}{t+\ln(t)} \right\}_{t=1}^{\infty}$

(b) $\left\{ \frac{n}{n+2} \right\}_{n=1}^{\infty}$

(c) $\left\{ \frac{n!}{e^{n^2}} \right\}_{n=1}^{\infty}$

(d) $\left\{ \frac{k!}{4^k k^4} \right\}_{k=1}^{\infty}$

(e) $\left\{ \frac{(n!)^2}{(2n)!} \right\}_{n=1}^{\infty}$

(f) $\left\{ \ln \left(1 - \frac{1}{k} \right) \right\}_{k=1}^{\infty}$

6. For each of the following sums, find a closed form for it and find its limit, if it exists, as $n \rightarrow \infty$.

(a) $\frac{1}{n} + \frac{2}{n} + \frac{4}{n} + \dots + \frac{2^{n-1}}{n}$

(b) $\frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \frac{1}{4 \cdot 6} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{n(n+2)}$

7. Suppose we have a 400 gallon tank half-full of water with 20 lbs of dissolved salt. At time $t = 0$, we begin to pour pure water into the tank at a rate of 35 gallons per minute while simultaneously draining water from the tank at a rate of 10 gallons per minute. How much salt is remaining in the tank at moment it is full?

8. Evaluate these sums:

(a) $\sum_{k=1}^{\infty} \ln \left(\frac{5k+3}{5k-2} \right)$

(b) $\sum_{n=6}^{\infty} 2 \left(-\frac{1}{2} \right)^n$

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^{2n}}$

(d) $1 + 5 - 7 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \dots$

(e) $\sum_{k=3}^{\infty} \frac{1}{k^2 - 2k}$