Review Sheet for First Exam

Mathematics 2300

September 20, 2006

The exam will cover Sections 6.9, 1.8, 7.4, 7.5, 7.6, and 7.7. In addition there will be some problems on substitutions (6.3 and 6.8).

No calculators of any kind will be allowed.

Formulas to remember:

- Standard indefinite integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C \qquad \int e^x\,dx = e^x + C$$

$$\int \cos x\,dx = \sin x + C \qquad \int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \sec x \tan x\,dx = \sec x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

- The Fundamental Theorem of Calculus:

$$\text{If } f \text{ is continuous at } x, \text{ then } \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

$$\text{If } f' \text{ is continuous in } [a,b], \text{ then } \int_a^b f'(x)\,dx = f(b) - f(a).$$

Also understand how to use the chain rule here to compute, e.g.,

$$\frac{d}{dx}\int_x^{x^2} f(t)\,dt.$$

- Length of a parametric curve: if $x'(t)$ and $y'(t)$ are continuous, then

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

  Understand how to derive the formula for length of $y = f(x)$ by using the standard parametrization $x = t$, $y = f(t)$. (Similarly if $x = f(y)$, use $x = f(t), y = t$.) Don't memorize three different formulas for arc length!

- Surface area of a parametric curve revolved around the y-axis: if $x'(t)$ and $y'(t)$ are continuous, then

$$S = \int_a^b 2\pi x(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

  Revolved around the x-axis:

$$S = \int_a^b 2\pi y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

  Remember that what appears outside the square root is the circumference of each circle; that's why it's $2\pi x(t)$ if revolved around the y-axis and $2\pi y(t)$ if revolved around the x-axis. As with arc length, do not memorize more than one formula for this; derive any other formula from this.

- Average value: if f is continuous, then

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\, dx$$

- Work:

$$W = \int_a^b F(x)\, dx$$

Be able to:

- Recognize where substitution can be used and compute indefinite integrals. Also be able to change limits of a definite integral after substitution.
- Graph a parametric curve $x(t), y(t)$ using one of the standard techniques:
  - In simple cases, solve for y in terms of x or x in terms of y to get a recognized graph.
  - In more complicated cases, plot points for particular t-values.

- Give a parametrization for simple curves like line segments, circles, and ellipses. Also be able to pass back and forth between explicit forms like $y = f(x)$ and parametrizations $x = t, y = f(t)$. This will enable you to reduce all arc length and surface area problems to the parametric form as mentioned above.

- Identify the orientation of a curve, how much of it is traversed, and how many times it is traversed.

- Identify the self-intersections of simple curves.

- Set up integrals for arc length, simplifying and identifying perfect squares to evaluate them. (These problems require great care in algebra, "foil"ing and such.) Be able to estimate arc length of a curve using approximation by segments.

- Sketch surfaces of revolution around the x-axis or y-axis. Even if you're not asked, this ability will help you remember the formula for surface area, since you can identify the circles and use the circumference mnemonic.

- Explain why hypotheses such as continuity are necessary in the formulas given above, such as for the Fundamental Theorem of Calculus. (For example, if f is a step function, then f is not the integral of its derivative.)

Don't do this! (Yes, it is *very* common!)
"The law of universal linearity":

- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

- $(a+b)^2 \neq a^2 + b^2$

- etc.

If you're not sure whether your algebra is legitimate, check a special case. For example, if you think $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$, plugging in $a = 1$ and $b = 4$ gives $\sqrt{5} = 3$ which is certainly wrong.

People tend to make these mistakes often when they get desperate to make an arc length or a surface area integral work.

Textbook review problems:

Make sure you know how to do all of these. The Chapter Review problems are more straightforward, while the Section problems are conceptual problems that are important to understand.

- Section 6.9: 27–28, 39, 44.
- Chapter 6 Review: 13–16, 33–34, 57–64, 66abc, 67–69, 93–103.
- Section 1.8: 29, 37, 39.
- Chapter 1 Review: 45, 46a, 47. (Do not use a graphing calculator.)
- Section 7.4: 22–23, 30.
- Section 7.5: 21, 23–26.
- Section 7.6: 19, 29–30.
- Section 7.7: 5.
- Chapter 7 Review: 13–18, 20.

Some other conceptual problems.

1. Suppose the length of $y = f(x)$ from $x = 0$ to $x = a$ is given by

$$L(a) = \ln\left(\frac{1+\sin a}{\cos a}\right),$$

for any value of $a > 0$. Find a formula for $f(x)$.

2. What is wrong with the following argument? Let

$$I = \int 2\sin x \cos x \, dx.$$

If we let $u = \sin x$ then $I = \sin^2 x + C$. However if we let $u = \cos x$ then $I = -\cos^2 x + C$. Therefore

$$\begin{aligned} \sin^2 x + C &= -\cos^2 x + C \\ \sin^2 x &= -\cos^2 x \\ \sin^2 x + \cos^2 x &= 0 \\ 1 &= 0. \end{aligned}$$

3. Describe a curve with infinite arc length.