**MATH 8174: Assignment 1**

1. Let $G$ be an abelian group. Show that every irreducible representation of $G$ over $\mathbb{C}$ is one-dimensional.

2. Suppose that $V$ is a one-dimensional representation of $G$. Show that every element of the commutator subgroup $G' = [G, G]$ acts as the identity matrix on $V$.

3. By considering the four long diagonals of a cube, prove that the group of rotations of a cube is isomorphic to the symmetric group $S_4$. Write down matrices for the rotations corresponding to the permutations (12) and (1234).

4. Given any prime $p$, find an example of a two-dimensional representation of some finite group over some field of characteristic $p$ that is indecomposable but not irreducible. Find also an example of such a representation in characteristic zero for an infinite group. (Hint: try the group of integers.)

5. The group of rotations of a regular dodecahedron is isomorphic to the alternating group $A_5$. Use this to prove that the group $A_5$ has an irreducible 3-dimensional representation over $\mathbb{C}$. (You do not need to find explicit matrices. In fact, please don't.)