1. Let $V$ be a 5-dimensional vector space over a field $F$. Prove (quoting any standard results that you need) that the exterior algebra $\wedge(V)$ has dimension 32 as an $F$-vector space.

2. Let 
\[
A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}
\]
be elements of $M_2(\mathbb{Q})$. Prove that there is no $2 \times 2$ invertible matrix $P$ with entries from $\mathbb{Q}$ such that $P^{-1}AP = B$. Is there a $2 \times 2$ invertible matrix with entries from $\mathbb{C}$ such that $P^{-1}AP = B$?

3. Let $F = \mathbb{Q}$ and $K = \mathbb{Q}(\sqrt{5}, i)$ be subfields of $\mathbb{C}$. Find the degree of $K$ over $F$, and write down a basis for $K$ as an $F$-vector space. Is $K$ algebraic over $F$?