

1. (10) Express the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$$

both as a product of disjoint cycles, and as a product of transpositions.

2. (20) Classify all abelian groups of order 500, up to isomorphism.

3. (50) There is a certain group of order 120 known as $SL(2, 5)$. You may assume the following facts about G : (a) G has precisely three normal subgroups: itself, the trivial subgroup, and its center, $Z(G)$; (b) $G/Z(G)$ is isomorphic to A_5 , the alternating group on 5 letters.

(i) Prove that $Z(G)$ has order 2.

(ii) Prove that G is not simple.

(iii) Prove that the commutator subgroup $C(G) = G'$ is equal to the whole of G .

(iv) Show that G cannot have a subgroup of order 60.

(v) Prove that G is not isomorphic to S_5 .

(vi) Prove that there exists a surjective homomorphism $\phi : G \rightarrow A_5$.

4. (20) True or False. Mark with a “T” or an “F,” and provide a brief explanation (a couple of lines), for each part.

(i) _____ If $C(G) = \{e\}$ then $Z(G) = G$. (Here, $C(G)$ is the commutator subgroup of G and $Z(G)$ is the center.)

(ii) _____ If $C(G) = G$ then $Z(G) = \{e\}$.

(iii) _____ If G is a group of order n and k divides n , then G has an element of order k .

(iv) _____ If G is a group of order n and G has an element of order k , then k divides n .

(v) _____ If H is a subgroup of G and H is abelian, then H is normal in G .

(vi) _____ If H is a subgroup of G and G is abelian, then H is normal in G .

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Mathematics 3140, Section 1: Second In-Class Exam

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Problem	Points	Score
1	10	
2	20	
3	50	
4	20	
Total	100	