

1. (40) Let $S = \mathbb{R} \setminus \{-1\}$ be the set of all real numbers excluding -1 . You may assume without proof that $(S, *)$ forms a group, where we define $a * b$ by

$$a * b = ab + a + b.$$

Let \mathbb{R}^* denote the group of nonzero real numbers under the ordinary operation of multiplication.

- (i) Show that the function $\phi : \mathbb{R}^* \rightarrow S$ defined by $\phi(r) = r - 1$ is an isomorphism. (You do not need to define the term “isomorphism”.)

- (ii) Show that the subset $H = \mathbb{Q} \setminus \{-1\}$ consisting of all the rational members of S forms a subgroup of S .

2. (25)

(i) List the elements in the subgroup H of \mathbb{Z}_{24} generated by the subset $\{6, 15\}$.

(ii) Is H isomorphic to the additive group \mathbb{Z}_n of integers modulo n for some $n > 1$?
If so, find n and a generator for H . If not, explain why not.

3. (15)

(i) Find the order of the symmetric group on 12 letters, S_{12} . (You may leave your answer as a formula.)

(ii) The group of all rotations that fix a regular tetrahedron (under composition of functions) is a certain group of order 12, known as A_4 . Explain why S_{12} must have a subgroup isomorphic to A_4 .

4. (20) True or False. Mark with a “T” or an “F,” and provide a brief explanation (a couple of lines), for each part.

(i) _____ If $(G, *)$ is a nonabelian group and a, b are two particular elements of G , then it must be the case that $a * b \neq b * a$.

(ii) _____ If $(G, *)$ is an abelian group and a, b are two particular elements of G , then it must be the case that $a * b = b * a$.

(iii) _____ A subgroup of a nonabelian group cannot be cyclic.

(iv) _____ A subgroup of a cyclic group cannot be nonabelian.

(v) _____ If G is a cyclic group, then there cannot be more than two different elements $a \in G$ for which $G = \langle a \rangle$.

(vi) _____ If G is an infinite cyclic group, then there cannot be more than two different elements $a \in G$ for which $G = \langle a \rangle$.

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Mathematics 3140, Section 1: First In-Class Exam

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Problem	Points	Score
1	40	
2	25	
3	15	
4	20	
Total	100	