Math 6230 Homework #7
due Friday, November 6

1. Read all of Section 13 (Vector fields and differential equations) of "Vector Calculus for Differential Geometry."

2. Find the flow Φ_t of the vector field $U = (x-y)\,\frac{\partial}{\partial x}\big|_{(x,y)} + (-x+y)\,\frac{\partial}{\partial y}\big|_{(x,y)}$ on $\mathbb{R}^2$.

3. For the vector field $V = (x^2+3xy+2y^2)\,\frac{\partial}{\partial x}\big|_{(x,y)} + (2x^2+3xy+y^2)\,\frac{\partial}{\partial y}\big|_{(x,y)}$, show that $[U,V] = 0$.

4. Compute directly the vector field $(\Phi_t)_*(V)$ and show that it's equal to V.

5. Define new coordinates (u,v) on $\mathbb{R}^2$ by $(x,y) = \Phi_u(v,0)$, where Φ_t is the flow of U that you found in problem 1. Compute V explicitly in coordinates (u,v).

6. Find the flow Ψ_t of V explicitly in (u,v) coordinates.

7. Verify explicitly in (u,v) coordinates that $\Psi_s \circ \Phi_t = \Phi_t \circ \Psi_s$ for any s and t.